Optimality and Natural Selection in Markets

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Abstract

Evolutionary arguments are often used to justify the fundamental behavioral postulates of competive equilibrium. Economists such as Milton Friedman have argued that natural selection favors profit maximizing firms over firms engaging in other behaviors. Consequently, producer efficiency, and therefore Pareto efficiency, are justified on evolutionary grounds. We examine these claims in an evolutionary general equilibrium model. If the economic environment were held constant, profitable firms would grow and unprofitable firms would shrink. In the general equilibrium model, prices change as factor demands and output supply evolves. Without capital markets, when firms can grow only through retained earnings, our model verifies Friedman's claim that natural selection favors profit maximization. But we show through examples that this does not imply that equilibrium allocations converge over time to efficient allocations. Consequently, Koopmans critique of Friedman is correct. When capital markets are added, and firms grow by attracting investment, Friedman's claim may fail. In either model the long-run outcomes of evolutionary market models are not well described by conventional General Equilibrium analysis with profit maximizing firms.

1. Introduction

Many theoretical results in economics and much of its applied power derives from the assumption that firms maximize profits. But this assumption does not have the same standing as the parallel assumption of preference maximization by consumers. Why should we assume that firms are run to maximize profits rather than something else, or for that matter, why should we assume that they maximize anything at all? One justification for profit maximizing firms is that non-maximizing firms will be driven from the market. We call this the “market selection hypothesis”. Another justification is that owners/shareholders want the firm to maximize profits (and presumably know how to do this and can enforce this discipline on the firm). This argument two relies on a market selection argument: inept owners can be profitably bought out by more efficiency-minded entrepreneurs. In this paper we study the market selection hypothesis and its consequences for general equilibrium analysis. We also investigate selection for owners-shareholders who can recognize and favor profit maximization over those who invest according to different criteria.

The best-known market selection defense of the profit maximization assumption was offered by Milton Friedman, who argued (Friedman, 1953, p. 22): “Whenever this determinant (of business behavior) happens to lead to behavior consistent with rational and informed maximization of returns, the business will prosper and acquire resources with which to expand; whenever it does not the business will tend to lose resources and can be kept in existence only by the addition of resources from the outside. The process of natural selection thus helps to validate the hypothesis (of profit maximization) or, rather, given natural selection, acceptance of the hypothesis can be based largely on the judgment that it summarizes appropriately the conditions for survival.” Alchian (1950) made similar arguments, as did Enke (1950) who wrote “In these instances the economist can make aggregate predictions *as if* each and every firm knew how to secure maximum long-run profits.” The intuition offered by Alchain, Enke and Friedman is that eventually capital markets will drive out firms that do not maximize profits.

Winter (1964, 1971) and Nelson and Winter (1982) make a different argument for the market selection hypothesis. A simple version of their argument is that the retained earnings of profit maximizers will grow fastest and thus they will come to dominate the market. Nelson and Winter construct a partial equilibrium model in which the “as if” hypothesis of profit maximization describes the long run steady state behavior of firms. In their analysis, prices are fixed and all firms have access to the same technology. This leads to the existence of a uniformly most fit firm (or a collection of identically-behaving fit firms) selected for by a retained earnings based investment dynamic.

The natural selection argument has its critics. Koopmans (1957, p. 140) argues that referencing an external dynamic process to support the validity of a key behavioral assumption is not really a satisfactory way to proceed: "But if this (natural selection) is the basis for our belief in profit maximization, then we should postulate that basis itself and not the profit maximization which it implies in certain circumstances." Nelson and Winter (1982, p. 58.) also understand that the coevolution of firm behavior and the economic environment could pose problems for the evolutionary defense of profit maximization. Among the “less obvious snags for evolutionary arguments that aim to provide a prop for orthodoxy” is “that the relative profitability ranking of decision rules may not be invariant with respect to market conditions.” However, there is no extant general equilibrium analysis of the consequences of replacing static profit maximization with a selection dynamic.

We construct a sequential-equilibrium market-clearing model in which a retained earnings dynamic, much like that discussed by Nelson and Winter, drives the scales of firm operation. The model is consistent with standard general equilibrium analysis in that rest points of the selection process are competitive equilibria, and the resulting allocations are Pareto optima. The questions we ask have to do with the attainment of the rest points. Starting from arbitrary initial conditions, will profit-maximizing firms be selected for, and will optimal allocations be achieved?

The answers to these questions demonstrate that Koopmans’ concern is justified. We show that defending profit maximization on natural selection grounds so as to be able to invoke the usual competitive analysis and assert that market outcomes are Pareto efficient is not a satisfactory way to proceed. We find that markets do favor profit-maximizing firms, but that producer efficient outcomes may nonetheless fail to emerge. The fact that markets favor profit maximization does not entail producer efficiency (much less so Pareto efficiency) because the selection process may never settle down and away from the rest points of the selection process, the competitive equilibria, prices may not lead to efficient coordination of firms’ activities. The weak link in the natural selection justification for the normative properties of competitive markets is not the behavioral hypothesis of profit maximization but the implication from profit maximization to Pareto optimality.

To ask if Friedman’s capital markets justification works we add capital markets to our model. If all investors have rational expectations then the addition of a market for one period investments is sufficient to generate dynamically complete markets. Equilibrium outcomes are thus Pareto optimal and no selection occurs or is needed. But if expectations are heterogeneous, then markets are dynamically incomplete. Equilibrium outcomes need not be optimal and we show that the market need not select for investors with rational expectations. We will see that in a certain sense the capital-market model is less well-behaved than the market model in which capital is reallocated only through the retained earnings dynamic.

This study of the connection between “economic fitness” and profit maximization is related to our earlier work on the market selection hypothesis in financial asset markets (Blume and Easley, (1992)) as well as the work by Sandroni (1997), which relies more heavily, as we do here, on optimization. The model described here is richer, however, despite the lack of stochastic shocks, because the real effects of investment decisions make the "fitness landscape" decidedly more complex than the concave hill of Blume and Easley and Sandroni. In this model we ask which expectations survive, which firms survive, and whether constrained equilibrium paths are asymptotically competitive.

In hindsight the negative answers that we have obtained are not surprising. In order to sensibly ask questions about evolution the market structure must be incomplete. So the question is really whether natural selection can substitute for complete markets. Of course, the incomplete markets equilibrium will not be a complete markets equilibrium from the start. But the natural selection conjecture is that from some interesting set of initial conditions (describing firms' capital or heterogeneous investors' wealths) the incomplete markets equilibrium converges to a complete markets equilibrium. Given how little structure incomplete markets equilibria have the conjecture seems incredible.

In the next section we lay out the basic equilibrium model. Firms are owned by capitalists who choose how much of the firm's revenue to leave in the firm as retained earnings and how much to consume. Production takes time and firms input purchases must be financed with their retained earnings. Thus we have a cash in advance constraint and no external market for financial capital. Equilibria in this model are called constrained equilibria. Section 3 demonstrates that the financial capital dynamic induced by constrained equilibrium selects for profit-maximizing firms over firms following other behavioral rules. The heart of the paper is in the following two sections. In section 4, we examine the connections between our constrained equilibria and competitive equilibria. We show that if a constrained equilibrium converges the limit is a competitive equilibrium and thus is Pareto optimal. In section 5, a series of examples demonstrates how constrained equilibria may fail to converge. The constrained equilibrium path in these examples can exhibit cycles as well as chaotic behavior. In section 6 a market for financial capital is added to the model. If investors have rational expectations then the market is dynamically complete and equilibrium outcomes are efficient. But we show the market need not select for rational investors in such a way that efficiency is attained. We offer our conclusions in section 7. All proofs are contained in the Appendix.

2. The Model

This section describes the basic conventions of the model and intertemporal equilibrium without markets for financial capital. Time is discrete, and is indexed by $t = 1, 2, \ldots$. At each date, the economy has J commodities, and date t prices are non-negative vectors $p_t \in \mathbf{R}^{\mathbf{J}}_{+}$. There are two types of infinitely-lived consumers: "workers" and "capitalists". Workers are indexed by $i = 1, \ldots, I$ and have stationary endowments $e^i \in \mathbf{R}^{\mathbf{J}}_{+}/\{0\}$ in each period. Capitalists are indexed by $h = 1, \ldots, H$ and own firms. The consumption set for both types is some non-negative orthant $C \subset \mathbf{R}^{\mathbf{J}}_{+}$. All consumers have perfect foresight.

Worker i has utility function $U_i(c) = \sum_{t=1}^{\infty} \beta_i^t u_i(c_t)$ over infinite consumption streams. Capitalist h has utility function $U_h(c) = \sum_{t=1}^{\infty} \beta_h^t u_h(c_t)$ over infinite consumption streams. The discount factors, β_i and β_h, are nonnegative and less than one. The one-period reward functions, $u_i(\cdot)$ and $u_h(\cdot)$, are strictly concave, C^2 and differentially strictly monotonic on the consumption set C. In addition, we make the usual assumption about indifference not transversally cutting the boundary of the consumption set.

Assumption I: For every consumer, capitalist or worker, and any sequence $\{c^n\}_{n=1}^{\infty}$ of consumption bundles such that for good j, $c_j^n \to 0$, $D_j u(c^n) \to +\infty$ for all consumption goods j.

Capitalist h owns firm h.[1] Firms turn today's inputs into outputs available tomorrow. The technology for firm h is described by a production possibility set $T^h \subset \mathbf{R}^{\mathbf{J}}$. The sets T^h are closed convex cones, that is, technology is convex and exhibits constant returns to scale. We assume that each firm h has a uniquely specified list of commodities that can be used as inputs and outputs. For firm h any input-output vector $\omega^h \in T^h$ can be written $\omega^h = (\omega^{h-}, \omega^{h+})$, where $\omega^{h-} \leq 0$ is the vector of inputs and $\omega^{h+} \geq 0$ is the vector of outputs. Our dynamics are driven by the assumption that production takes time. Inputs ω_t^{h-} available at date t are used to produce outputs ω_{t+1}^{h+} at date $t+1$. For a given price vector p, we will let p^{h+} and p^{h-} denote the vector of prices for firm h's outputs and inputs, respectively. So $p_t^{h-}\omega_t^{h-}$ is the value of firm h's date t inputs and $p_{t+1}^{h+}\omega_{t+1}^{h+}$ is the value of firm h's date $t+1$ outputs.

2.1. Constrained Equilibrium

The set of available intertemporal contracts is constrained. Workers have no opportunities for lending or borrowing across different dates. Capitalists can transfer resources through time, but only through their production technology. In each period capitalists receive their firm's revenue. They decide how much to spend on current consumption, and how much to invest in their firm to generate tomorrow's revenues. We assume that the firm's input purchases must be financed with this investment of financial capital. Thus we have a cash-in-advance constraint on firms. [2] Specifically, retained earnings, or financial capital, is used to purchase inputs at date t. These inputs generate output, and thus revenue, at date $t+1$. The portion of this revenue that is retained in the firm becomes its new financial capital. The economy is initialized by endowing each capitalist with a stock of outputs $\omega_1^{h+} > 0$, which can be traded in the first period for inputs and other consumption goods.

We test for the emergence of profit maximization, and therefore we need to allow for a variety of behaviors by capitalists. Capitalist h follows a decision rule:

$$(\omega_t^{h-}, \omega_{t+1}^{h+}) \in d^h(p_t, p_{t+1}, y_t^h)$$

where p_t and p_{t+1} are the prices (for all goods) firm h faces at dates t and $t+1$, respectively, and y_t^h is the amount firm h has to spend on inputs at date t. Decision rules have to satisfy three properties:

1. Production must be feasible: $d^h(p, q, y) \in T^h$.
2. The firm's budget constraint must be met: $p^{h-} \cdot d^{h-}(p, q, y) = y$.
3. The decision rule is upper hemi-continuous.

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[1] We do not consider multiple owners of firms. What is important for our analysis is that the owner(s) of a firm want it to maximize profits. With a single owner, perfect competition and a deterministic world this is clear. With multiple owners we would also need to consider the mechanism determining payouts.

[2] Within our model, this cash in advance constraint is necessary to have financial capital play any role. There may be other interesting ways to model the evolution of firms, such as durable and nonreversible investment in physical capital, but we focus on financial capital.

One such rule is constrained profit maximization:

$$\begin{array}{rl} \max & q^{h+} \cdot \omega^{h+} - q^{h-} \cdot \omega^{h-} \\ \text{s.t.} & w \in T^h \\ & p^{h-} \cdot \omega^{h-} = y^h \end{array}$$

We denote this special decision rule by $D^h(p, q, y)$. Note that it is equivalent to revenue maximization subject to the operating capital constraint.

The constrained profit maximization decision rule exhibits *homogeneity.* If prices and revenues are rescaled so as to leave the firm's budget set unchanged, and output prices are rescaled so that relative prices of outputs do not change, then optimal production plans do not change. We assume that each firm uses a homogenous decision rule.

Definition 2.1: A decision rule $d(p, q, y)$ is *homogeneous* if for all positive scalars α and β, and all prices, price expectations and revenues p, q and y, $d(\alpha p, \beta q, \alpha y) = d(p, q, y)$.

Equilibrium in this model is a sequence of prices, consumption bundles and production plans such that consumers maximize utility subject to various constraints, and such that the allocation is feasible. For each worker, the constraints are the single-period budget constraints. For each capitalist, the constraints are budget constraints involving the allocation of resources between consumption and production, and the decision rule. We call equilibrium with behavior as described above *constrained equilibrium.* Formally,

Definition 2.2: A *constrained equilibrium* is a sequence $\left(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H\right)_{t=1}^\infty$ with $p_t^* \in \mathbf{R}_+^\mathbf{J}/\{0\}$ such that

1. For all workers i, $\{x_t^{i*}\}_{t=1}^\infty$ solves

$$\begin{array}{rl} \max\limits_x & \sum\limits_t \beta_i^t u_i(x_t) \\ \text{s.t.} & p_t^* \cdot (x_t - e^i) \leq 0 \text{ for all } t, \\ & x \in C. \end{array}$$

2. For all capitalists h, $\{x_t^{h*}, \omega_t^{h*}\}_{t=1}^\infty$ solves

$$\begin{array}{rl} \max\limits_{x,w} & \sum\limits_{t=1}^\infty \beta_h^t u_h(x_t) \\ \text{s.t.} & p_t^* \cdot (x_t - \omega_t^{h*+} - \omega_t^{h*-}) \leq 0, \\ & (\omega_t^{h*-}, \omega_{t+1}^{h*+}) \in d^h(p_t^*, p_{t+1}^*, p_t^* \cdot \omega_t^{h*-}) \text{ for all } t, \\ & \text{and} \quad x \in C. \end{array}$$

3. At every date t, $\sum_i x_t^{i*} + \sum_h x_t^{h*} + \sum_h \omega_t^{h*} - \sum_i e^i = 0$,
   where $(w_1^{h*+})_{h=1}^H$ is given.

In a standard competitive equilibrium, a consequence of the 0-degree homogeneity of demand and supply in prices is that the aggregate price level is indeterminate. Constrained equilibrium exhibits more price-level indeterminacy because consumers and firms are not free to take advantage of arbitrary relative intertemporal prices. In an economy with homogeneous decision rules, constrained equilibrium determines relative prices only among commodities available at the same date. The price level is indeterminate, period by period.

Lemma 2.1: Suppose firm decision rules are homogeneous. If

$$\left(p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H\right)_{t\geq 1}$$

is a constrained equilibrium and $(\lambda_t)_{t\geq 1}$ is a sequence of strictly positive scalars, then

$$\left(\lambda_t p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H\right)_{t\geq 1}$$

is also a constrained equilibrium.

Constrained equilibria have an important recursive property, whose proof is an immediate consequence of the definition.

Lemma 2.2: If $\left(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H\right)_{t=1}^\infty$ is a constrained equilibrium, then so is $\left(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H\right)_{t=T}^\infty$ for any T.

2.2. Competitive Equilibrium

We are interested in the relationship between constrained equilibria and competitive equilibria.

Definition 2.3: A *competitive equilibrium* is a sequence $\left(q_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H\right)_{t=1}^\infty$ with $q_t^* \in \mathbf{R}_+^{\mathbf{J}}/\{0\}$ such that

1. For all workers i, $\{x_t^{i*}\}_{t=1}^\infty$ solves

$$\begin{aligned} \max_x \quad & \sum_t \beta_i^t u_i(x_t) \\ \text{s.t.} \quad & \sum_{t=1}^\infty q_t^* \cdot (x_t - e^i) \leq 0, \\ & x \in C. \end{aligned}$$

2. For all capitalists h, $\{x_t^{h*}, \omega_t^{h*}\}_{t=1}^\infty$ solves

$$\max_{x,w} \quad \sum_{t=1}^\infty \beta_h^t u_h(x_t)$$

$$\text{s.t.} \quad \sum_{t=1}^{\infty} q_t^* \cdot (x_t - \omega_t^{h+} - \omega_t^{h-}) \leq 0$$
$$x \in C.$$
$$(\omega_t^{h-}, \omega_{t+1}^{h+}) \in T^h \quad \text{for all } t.$$

3. At every date t, $\sum_i x_t^{i*} + \sum_h x_t^{h*} + \sum_h \omega_t^{h*} - \sum_i e^i = 0$, where $(\omega_1^{h*+})_{h=1}^H$ is given.

The competitive equilibrium described here is equivalent to a competitive equilibrium in a private-ownership economy in which each capitalist owns all of his own firm and maximizes profit.

Lemma 2.3: Suppose that $\left(q_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H\right)_{t=1}^{\infty}$ is a competitive equilibrium, and let $\pi_t^{h*} = q_{t+1}^{*h+}\omega_{t+1}^{h*+} - q_t^{*h-}\omega_t^{h*-}$. Then:

1. Each firm h maximizes profits: $\pi_t^{h*} \geq q_{t+1}^{*h+}w^+ - q_t^{*h-}w^-$ for all $(w^-, w^+) \in T^h$.
2. Each firm h earns 0 profits: $\pi^{h*} = 0$.

The definition of competitive equilibrium presupposes the existence of a market structure sufficient to transfer wealth across dates and firms. Constrained equilibrium presupposes that the market structure is inadequate for this task. Our interest is in whether the dynamics induced by constrained equilibrium eventually compensates for the lack of complete markets.

3. Selection for Profit Maximizers

The first question to ask about the dynamics induced by constrained equilibria is whether profit maximizing firms are selected for; or, more carefully, whether non-profit maximizing firms are driven out of the market. Writers such as Alchian (1950) and Friedman (1953) have defended the profit-maximization hypothesis using evolutionary arguments. Winter (1971) and Nelson and Winter (1982) formalized this intuition in a dynamic model that shares some features with our model. In particular, in Nelson and Winter's work profitable firms grow and unprofitable ones shrink. In Nelson and Winter's analysis, prices are fixed and all firms have access to the same technology. This leads to the existence of a uniformly most fit firm (or a collection of fit firms behaving identically) which is selected for by an investment dynamic similar to ours. But in our economy, prices are endogenous and firms do not all have access to identical technologies.

We begin with a general result about the fate of two capitalists with differing firm decision rules, utility functions and discount factors. The key to the result is the relationship between the capitalists discounted marginal rates of return on investment. Along any constrained equilibrium path, each capitalist sets his marginal rate of substitution between expenditure on consumption at dates t and $t+1$ equal to his discounted marginal rate of return on investment at date t. More generally, the marginal rate of substitution between expenditure on consumption at dates 1 and T will be equal to the product of discounted marginal rates of return on investment from date 1 to date

T. Suppose capitalist h has a uniformly, over time, larger discounted rate of return on investment than does capitalist k; that is, h faces a more attractive, from his pint of view, investment opportunity at each date than does k. Then h's marginal rate substitution between consumption at dates 1 and T must grow exponentially relative to k's. Consumption is bounded above, so marginal rates of substitution are bounded above. Thus k's marginal rate of substitution must converge to 0. That is, the marginal utility of income must diverge for k. So k's consumption and the financial capital of the firm owned by k must converge to 0. Of course, k is choosing this path, but nonetheless he is being driven out of the market by h.

The intuition above uses the Euler equation to describe each capitalist's optimal path. For this to be legitimate we need to be sure that the Euler equation is well defined and necessary. Let $\rho^h(p^{h-}, p^{h+}, y)$ denote the revenues of firm h at input prices p^{h-}, output prices p^{h+} and expenditure level y.

Assumption R: For every firm h,

1. The partial derivative $\rho^h_y(p^{h-}, p^{h+}, y)$ exists.
2. $\rho^h_y(p^{h-}, p^{h+}, 0) \neq 0$.

The first condition says that marginal rates of return on investment are well defined. The second assumption, along with our Inada condition on utility functions (Assumption I), rules out boundary solutions with 0 investment in finite time.

Theorem 3.1: Suppose that Assumptions I and R hold. In any constrained equilibrium $\left(p_t, (x^i_t)^I_{i=1}, (x^h_t, \omega^h_t)^H_{h=1}\right)^\infty_{t=1}$, and for any capitalists h and k with discount factors β_h and β_k:

$$\prod_{\tau=1}^{t-1} \frac{\beta_k \rho^k_{y\tau}}{\beta_h \rho^h_{y\tau}} \to 0 \quad \text{implies} \quad \lim_t \frac{p_t \omega^{k-}_t}{\sum_i p_t \omega^{i-}_t} = 0 \quad \text{and} \quad c^k_t \to 0.$$

Theorem 3.1 provides a general characterization of the market selection process. Which capitalists-firms survive depends on discount factors and marginal rates of return; but it does not depend on one-period utility functions. The only feature of utility functions that matters for the result is that marginal utility of consumption diverges as consumption goes to 0.

With a bit of structure on firm's revenue functions Theorem 3.1 has implications for the survival of constrained profit maximizers. Along any equilibrium path, let $r^h_t = \rho(p^{h-}_t, p^{h+}_{t+1}, p^t \omega^{h-}_t)/p^{h-}_t \omega^{h-}_t$ denote average return on investment in period t. Note that for constrained profit maximizing firms, $\rho^h_y(p^{h-}, p^{h+}, y)$ exists and is a constant, independent of y. Thus for a constrained maximizer $\rho^h_{yt} = r^h_t$ for all t. To insure that a maximizer drives out a non-maximizer with same discount factor we need to rule out increasing returns to investment by the non-maximizer.

Corollary 3.1: Suppose that Assumptions I and R hold, and that capitalist h maximizes constrained profits and capitalist k uses a decision rule such that $\rho^h_y(p^{h-}, p^{h+}, y)$ is non-increasing in

y and

$$\prod_{\tau=1}^{t-1} \frac{\beta_k r_\tau^k}{\beta_h r_\tau^h} \to 0.$$

Then the conclusions of Theorem 3.1 still hold.

If both capitalists have the same discount factor, then the profit maximizer, with the higher average return on investment, drives out the other firms. This confirms Winter's (1971) and Nelson and Winter's (1982) results in our economy. Notice that this works even if the two firms are in different industries, or have different technologies available to them. But if decision rules and discount factors are correlated in some funny way, then higher discount factors can compensate for inferior decision rules. The important role of discount factors in driving market selection is demonstrated in the following corollary, which shows that if two profit-maximizing capitalists have not too dissimilar long run rates of return, then discount factors determines who survives, independent of tastes.

Corollary 3.2: If Assumptions I and R hold, and if capitalists h and k are profit maximizers, and if

$$0 < \liminf_t \prod_{\tau=1}^{t-1} \frac{r_\tau^k}{r_\tau^h} \leq \limsup_t \prod_{\tau=1}^{t-1} \frac{r_\tau^k}{r_\tau^h} < \infty$$

and if $\beta_k/\beta_h < 1$, then the conclusions of Theorem 3.1 hold.

From Theorem 3.1 one might suspect that if the one firm's decision rule is less efficient than the "aggregate decision rule" of the other firms in the market, then the inefficient firm will be driven out and the production side of the economy would operate efficiently in the limit. This conclusion is incorrect however, as the following sections show.

4. Dynamics

The previous section shows that if, among all firms with the same technology, at least one firm belonging to a capitalist with the maximal discount factor maximizes profit then any survivor will be a profit maximizer. Or, even if none of the firms profit maximize, the market selects from among the firms with a given technology those firms which are most profitable. The question that we turn to now is whether the financial capital dynamic also insures that each industry operates efficiently. The questions of interest are: If several firms produce several goods from common inputs with differing technologies, does the market select for those firms which are most efficient? In particular, does the economy eventually operate on the production possibility frontier and does it eventually achieve a Pareto optimal allocation? If a new firm enters an industry with an efficient technology (one that expands the production possibility frontier in a relevant direction) will this firm flourish (or is it possible that it will be driven out by the retained earnings dynamic)?

The answers to these questions are "no" if there are no profit-maximizing firms, or if the profit-maximizing firms belong to capitalists with low discount factors. To see this consider two capitalists with the same technology and differing discount factors. If the capitalist with the low discount factor owns the profit-maximizing firm, and if the other firm has a sufficiently high (although not maximal) average rate of return, then according to Theorem 3.1 the profit-maximizing firm would disappear. In the limit the economy would not be operating on its production possibility frontier. To rule this phenomenon out, we assume for the remainder of the paper:

Assumption D: *i.* All consumers have a common discount factor β, and
ii. There is a set of available technologies $\{T_k\}_{k=1}^K$. For each technology T_k there is at least one capitalist h_k who maximizes constrained profit using technology T_k.

Unlike the analysis of the previous section the answers to the general equilibrium questions posed here depend on prices and thus on the evolution of constrained equilibria. To answer these questions we need to analyze the dynamics induced by constrained equilibrium in more detail. In particular the relationship between constrained and competitive equilibria is important.

Under rather general conditions, every competitive equilibrium is Pareto optimal. Also under rather general conditions, competitive equilibria will have turnpike properties. We will place sufficient structure on the economy that competitive equilibria are easily characterized as stationary after the first period. To guarantee this we need to assume that workers endowments are not consumer goods and inputs are not produced.

Assumption C: There is a partition of the set of commodities $\{1, \ldots, J\}$ into two sets $\mathbf{Inp}$ and $\mathbf{Con}$ such that

1. For all firms h, if $(w^{h-}, w^{h+}) \in T^h$ then $\omega^- \in \mathbf{R}^{\mathbf{Inp}}$ and $\omega^+ \in \mathbf{R}^{\mathbf{Con}}$.
2. For all i, $e^i \in \mathbf{R}^{\mathbf{Inp}}$.
3. $C = \mathbf{R}_+^{\mathbf{Con}}$.

Theorem 4.1: If Assumptions I, C and D hold, then every competitive equilibrium consumption path is stationary from period 2 on. That is, if $\left(q_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H\right)_{t=1}^\infty$ is a competitive equilibrium, then for each i and h, respectively, there are consumption bundles x^i and x^h such that $x_t^i = x^i$, $x_t^h = x^h$ for all $t \geq 2$.

Non-stationary competitive equilibrium production paths are possible because of our assumption of constant returns to scale. But the proof of Theorem 4.1 shows that every competitive consumption path can be supported by a competitive equilibrium in which production plans are stationary. We call such equilibria *stationary competitive equilibria*.

In a stationary competitive equilibrium, the financial capital (the amount spent on inputs) is constant and because the equilibrium is stationary workers do not save or borrow. If firms are given initial financial capital equal to the amount spent on inputs in the competitive equilibrium, then the competitive prices clear markets. Because the firms have zero profits at these prices, their financial capital is constant. Consequently this competitive equilibrium is a constrained equilibrium. This argument is summarized in the following theorem.

Theorem 4.2: A stationary competitive equilibrium is a constrained equilibrium for some assignment of initial outputs $(\omega_1^h)_{h=1}^H$.

The relevant question is whether constrained equilibria with other initial financial capital stocks converge to a competitive equilibrium. Competitive equilibria in our economy are Pareto optimal. So if a constrained equilibrium converges to a competitive equilibrium the limit equilibrium is optimal and thus stationary. A constrained equilibrium path is stationary if consumption paths are stationary, and if each firms share of total factor costs remains constant over time. We first show that any stationary constrained equilibrium with at least one constrained profit maximizer per technology active is competitive.

Definition 4.1: A constrained equilibrium $\left(p_t^*, (x_t^{i*})_{i=1}^I, (x_t^{h*}, \omega_t^{h*})_{h=1}^H\right)_{t=1}^\infty$ is *stationary* if there exists consumption bundles x^i and x^h for each worker and capitalist, respectively, and production plans w^h such that the following properties hold for all $t \geq 1$: For all workers, $x_t^i = x^i$, for all capitalists, $x_t^h = x^h$, and for all firms, $\omega_t^h = \omega^h$. A stationary constrained equilibrium is *interior* if for each technology k, $\omega^{h_k} \neq 0$. Finally, a stationary state is *locally stable* if for any initial outputs of the firms $(\omega_1^{h+})_{h=1}^H$ sufficiently close to $(\omega^h)_{h=1}^H$, there is a constrained equilibrium path such that workers consumptions converge to the respective x^i, capitalists consumptions converge to the respective x^h, and production plans converge to the respective ω^h.

Theorem 4.3: Suppose Assumptions I, C and D hold. The allocation resulting from any stationary and interior constrained equilibrium is a competitive allocation.

Not all stationary constrained equilibria are competitive. Suppose that there are two technologies, each used by exactly one capitalist, and that one of the capitalists is endowed with 0 initial output. This capitalist's firm can never grow, so the constrained equilibrium is stationary, but unless the non-producing firm's technology is redundant, this equilibrium is not optimal and therefore not competitive. Suppose however that the constrained equilibrium path is initially interior and converges to a stationary state in which some firm has zero financial capital and is thus inactive. This firm must be making loses along the way since it once had positive financial capital. If prices were continuous in financial capital stocks it would follow that the firm would make a loss if it operated at the limit prices. Thus the financial capital constraint would not be binding on such a firm. To insure the needed continuity we place an assumption on workers endowments that guarantees uniqueness of prices. With this assumption we show that every locally stable, stationary constrained equilibrium is competitive.

It will be convenient to assume that every constrained equilibrium is supported by a price vector that is unique up to the renormalization described by Lemma 2.1. As a consequence of our assumptions on preferences this already holds for consumption goods prices because each consumption bundle is supported by a unique budget line. We could use similar smoothness, curvature and boundary assumptions on production to guarantee the uniqueness of supporting input prices but our examples in Section 5 all involve piecewise linear production. The following non-degeneracy (ND) assumption has the same effect.

Assumption ND: The matrix of worker endowments

$$\begin{pmatrix} e_1 \\ \vdots \\ e_I \end{pmatrix}$$

has rank equal to $\mathbf{Inp}$, the number of inputs.

Given the workers' consumption bundles $(x^i)_{i=1}^I$ and consumer prices $p^{\mathbf{Con}}$, the workers' budget constraints must solve

$$p^{\mathbf{Inp}} \begin{pmatrix} e_1 \\ \vdots \\ e_I \end{pmatrix}^T = p^{\mathbf{Con}} \begin{pmatrix} x_1 \\ \vdots \\ x_I \end{pmatrix}^T .$$

Assumption ND implies that for each $(x^i)_{i=1}^I$ and $p^{\mathbf{Con}}$ there is a unique $p^{\mathbf{Inp}}$ which allows all the budget equations to be met.

Theorem 4.4: Suppose Assumptions I, C, D and ND hold. If a constrained equilibrium is locally stable, then the limit allocation is competitive.

5. Stability

We know from Theorems 4.3 and 4.4 that if a constrained financial equilibrium converges the limit allocation is competitive. These results support the argument that natural selection leads to optimality. But in this section we show that the conclusion, that natural selection in markets implies optimality, is not correct. It fails because the financial capital dynamic need not converge, and because non-steady-state behavior can be far from optimal. The following example shows that even in a standard economy with a unique competitive equilibrium, financial capital stocks need not converge. We find a limit cycle of revenues and a corresponding limit cycle of constrained equilibria, none of which are competitive equilibria.

In the examples in this section there are two consumption goods x and y, and a single input good z. We assume that all firms maximize profits subject to their expenditure constraints. Constrained equilibrium prices are normalized so that the value of aggregate input purchases in each period is 1. All consumers have utility functions on infinite consumption paths of the form

$$u(x, y) = \sum_{t=1}^{\infty} \beta^t \log(x_t^\rho + y_t^\rho)^{1/\rho}.$$

Both β and ρ are common to all consumers. Consequently, demand at each date aggregates. Furthermore, one-period indirect utility for a capitalist with income z is $\log z + \phi(p^{\mathbf{Con}})$, where ϕ depends upon ρ. Thus the intertemporal decision problems for capitalists are particularly simple. The solutions all require that capitalists invest a constant fraction β of their revenues in input purchases.

Example 5.1: There are two capitalists and one worker. The worker is endowed with good z which is used by the firms to produce x and y. Firm one produces 1 unit of x and 0.1 units of y at date $t+1$ for every unit of z that it purchases at date t; firm 2 produces 0.001 units of x and 1 unit of y at date $t+1$ for every unit of z that it purchases at date t.

For any ρ this economy has a unique competitive equilibrium with constant relative prices

$$p_z^* = 1, \quad p_x^* = 0.90009, \quad \text{and} \quad p_y^* = 0.9991$$

and quantities which depend on ρ.

In any constrained equilibrium, capitalists invest fraction β of their revenues in their firm and spend the remaining fraction on their consumption. Workers consume the entire value of their endowment. The demands by any consumer for goods x and y are

$$x = \frac{I}{p_x^{1-r}(p_x^r + p_y^r)} \qquad \text{and} \qquad y = \frac{I}{p_y^{1-r}(p_x^r + p_y^r)}$$

where $r = \rho/(\rho - 1)$ and I is the consumer's expenditure on consumption. These demands aggregate, and so at any date t,

$$\frac{p_{xt}}{p_{yt}} = \left(\frac{x_t}{y_t}\right)^{\rho-1}. \tag{5.1}$$

We normalize prices at each date so that expenditures on inputs always equal 1. Thus at date $t-1$, $R_{t-1}^1 + R_{t-1}^2 = 1$ and so firm h purchases share R_{t-1}^h of inputs. Consequently production of firm h is

$$(x_t^h, y_t^h) = \begin{cases} R_{t-1}^1(1, 0.1) & \text{if } h = 1, \\ R_{t-1}^2(0.001, 1) & \text{if } h = 2. \end{cases}$$

Recalling that fraction β of the revenues from the sale of these outputs will be used to purchase more input at date t we have,

$$R_t^h = \begin{cases} \beta(p_{xt} + 0.1 p_{yt})R_{t-1}^h & \text{if } h = 1, \\ \beta(0.001 p_{xt} + p_{yt})R_{t-1}^h & \text{if } h = 2. \end{cases}$$

Total expenditure on date t consumption has to equal the total wealth of the capitalists, for what the capitalists do not consume directly they transfer to the workers in return for inputs, and the workers spend this payment on consumption goods. With our normalization, total capitalist wealth must equal $1/\beta$. From equation (5.1) and the aggregate budget equation

$$p_{xt}x_t + p_{yt}y_t = \frac{1}{\beta},$$

so

$$p_{yt} = \frac{1}{\beta}\frac{y_t^{\rho-1}}{x_t^\rho + y_t^\rho} \qquad \text{and} \qquad p_{xy} = \frac{1}{\beta}\frac{x_t^{\rho-1}}{x_t^\rho + y_t^\rho}.$$

Consequently, the financial capital dynamic is

$$
\begin{aligned}
x_{t+1} &= R_t^1 + 0.001(1 - R_t^1) \\
y_{t+1} &= 0.1R_t^1 + (1 - R_t^1) \\
R_{t+1}^1 &= R_t^1 \left[\frac{x_{t+1}^{\rho-1} + 0.1y_{t+1}^{\rho-1}}{x_{t+1}^{\rho} + y_{t+1}^{\rho}} \right].
\end{aligned}
$$

We know from Theorem 4.3 that for each ρ this dynamic has exactly one interior steady state and that this steady state is the competitive equilibrium. If this steady state is locally stable then for any $0 < R_1^1 < 1$ the sequence of constrained equilibria converges to the competitive equilibrium. Otherwise, more complex limit behavior must occur. Calculation shows that at the steady state:

$$
\left| \frac{dR_{t+1}^1}{dR_t^1} \right| \begin{matrix} < \\ > \end{matrix} 1 \quad \text{as} \quad \rho \begin{matrix} > \\ < \end{matrix} -1.49.
$$

So as long as the consumption goods are not too strongly complementary the competitive equilibrium is locally stable. But if ρ is sufficiently small, less than -1.49, then the unique competitive equilibrium is unstable. □

FIG 1. HERE.

Figure 1 illustrates the map from R_t^1 to R_{t+1}^1 for $\rho = -3$. The instability of the steady state arises because if firm 1's purchasing power is a little too large, then the output of good x will be above its competitive equilibrium level. Because of the shape of the consumer's indifference curves this extra output of good x, and corresponding reduced output of good y, will reduce the market clearing price of good x so much that firm one experiences a large loss and earnings fall below the equilibrium level. But when firm one's purchasing power is low, the output of good x is reduced and it has a high price causing firm one's revenue to rise above the equilibrium level. When the goods are sufficiently complementary this cycle of profits and losses produces cycles in the levels of financial capital.

Figure 2 illustrates the behavior of limit financial capital stocks as a function of ρ. The data for this figure were generated by iterating the dynamic above starting from an initial financial capital for firm one of $R_1^1 = 0.5$. For each value of ρ the equilibrium equation system was iterated until either it was evident that a stable cycle had been reached or until it had been iterated $80,000$ times. For $\rho > -1.49$ the purchasing power of firm one converges to its steady state value and the limit allocation is competitive. For $-2.22 < \rho < -1.49$ a two-cycle emerges; for $-2.44 < \rho < -2.22$ a four-cycle emerges; and so on, generating a period-doubling cascade. For sufficiently negative values of ρ this map displays chaotic behavior with the limit purchasing power of firm 1 varying from about 0.2 to almost 1. As a result of this instability, the economy never achieves a Pareto optimal allocation.

FIG 2. HERE.

A firm caught in a two-cycle is making a loss in one period followed by an offsetting profit in the next. If there was a market for financial capital, and if investors had perfect foresight, they would never put their capital in firms that would be unprofitable. We consider the ability of capital markets to resolve this problem in Section 6. For now we note that Example 5.1 demonstrates that for some economies the internal capital market induced by having profitable firms grow and unprofitable ones shrink is not sufficient to achieve optimality.

In Example 5.1 no Pareto optimal allocation is ever achieved. Nonetheless production does take place on the boundary of the economy's aggregate production possibility frontier. Pareto optimality fails only because the "right" mix of commodities is never produced. With only two firms, inefficient production cannot occur as no matter how financial capital is allocated, the resulting allocation must be on the production possibility frontier. But with three or more firms even producer-efficiency can disappear. In the following example all efficient firms vanish.

Example 5.2: Now there are four firms: Firm 1 produces 1.0 x and $0.1y$ from 1 unit of z; firm 2 produces $0.05x$ and $1y$ from 1 unit of z; firm 3 produces $0.9x$ and $0.15y$ from 1 unit of z; and, firm 4 produces $0.3x$ and $0.7y$ from 1 unit of z. Calculation of the efficient frontier shows that the production processes used by firms 3 and 4 are dominated by combinations of those used by firms 1 and 2. Thus efficient production requires that only firms 1 and 2 operate. For any ρ there is a unique competitive equilibrium, in this equilibrium firms 3 and 4 do not produce and this equilibrium corresponds to a steady state of the dynamic with only firms 1 and 2 having positive financial capital.

FIG 3. HERE.

For sufficiently small ρ the steady state is unstable. Figure 3 shows the time path of financial capital for the four firms starting from an initial allocation of $R_1^1 = 0.4975, R_1^2 = 0.4975, R_1^3 = 0.0025, R_1^4 = .0025$. In this example $\rho = -11$. The graph shows a sequence of transitions from two efficient firms almost at the steady state, to two efficient firms and an inefficient firm almost in a two-cycle, to one efficient and one inefficient firm almost in a two cycle, to one efficient and two inefficient firms almost in a two cycle, to two inefficient firms almost in a two cycle, and finally to two inefficient firms in a four cycle. □

The result of Example 5.2 is particularly disturbing from the point of view of entry of efficient firms. Consider an economy in which only the two "inefficient" firms 3 and 4 exist. Example 5.2 shows that the financial capital dynamic for this economy has a stable four-cycle. Now suppose an entrepreneur discovers the technology of firm 1. This technology expands the aggregate production possibility set and would be used in any competitive equilibrium. If the entrepreneur begins with little financial capital, he will lose it. Actually, simulations show that even if he begins with a large initial financial capital, say $R_1^1 = 1/3$, he will lose it. The inefficient firms drive out the efficient firm.

6. Financial Markets

Capitalists would like to invest only in those firms which they expect will earn the highest rates of return. In the model analyzed in the previous sections capitalists do not have this opportunity. Now we add a market for investment in firms and a market for loans. We assume (for now) that all consumers have perfect foresight about future prices so that they make correct investment decisions.

Loans made by consumers at date t are denoted l_t^i and have a gross rate of return of R_{t+1} at date $t+1$. Loans are in zero net supply so the market clearing condition is that their sum across all workers and capitalists is zero. With access to consumption loans, both workers and capitalists can transfer income over time. To insure that the present discounted value of each consumers expenditures on consumption is no more than the present discounted value of his income we require that asymptotically the present value of loans is nonnegative.

Each capitalist also has the opportunity to invest his savings in any firm he chooses. Firms use this investment as they used the investment of their owners in the previous model: To purchase inputs today in order to produce output and thus revenue tomorrow. This revenue is paid out to the investors with each investor getting a share of the firm's revenue equal to the share of financial capital that he provided. Formally, at each date t capitalist h decides how much to spend on current consumption $p_t x_t^h$, how much to loan out l_t^h and how much to save for investment in firms s_t^h. The capitalist invests fraction $\alpha_{k\,t}^h$ of s_t^h in firm k at date t. Firm k's expenditures in period t are thus $y_t^k = \sum_h \alpha_{kt}^h s_t^h$. The rate of return between periods t and $t+1$ on this investment in firm k is $r_{k\,t}^* = p_{t+1}^{\mathbf{Con}*} \omega_{t+1}^{kt*+} / p_t^{\mathbf{Inp}*} \omega_t^{kt*-}$. So capitalist h will have income $\sum_k \alpha_{k\,t}^h s_t^h r_{k\,t}^* + R_{t+1} l_t^h$ in period $t+1$.

The definition of an equilibrium with financial markets is an extension of the definition of constrained equilibrium to include loans by consumers and investment by capitalists in other capitalist's firms.

Definition 6.1: A *rational expectations constrained financial equilibrium* is a sequence

$$\left(p_t^*, R_t^*, (x_t^{i*}, l_t^{i*})_{i=1}^I, (x_t^{h*}, \alpha_t^{h*}, s_t^{h*}, l_t^{h*})_{h=1}^H, (w_t^{k*})_{k=1}^K\right)_{t=1}^\infty$$

with $p_t \in \mathbf{R}_+^{\mathbf{J}}/\{0\}$ such that

1. For all workers i, $\{x_t^{i*}, l_t^{i*}\}_{t=1}^\infty$ solves

$$\begin{aligned} \max_{x,l} \quad & \sum_t \beta_i^t u_i(x_t) \\ \text{s.t.} \quad & \text{(i) } x \in C, \quad \liminf_t \frac{l_t}{\prod_{\tau=1}^t R_\tau^*} \geq 0 \quad \text{and,} \\ & \text{(ii) For all t:} \quad p_t^* x_t + l_t \leq m_t, \\ & m_t = p_t^* e + R_t^* l_{t-1}. \end{aligned}$$

2. For all capitalists h, $\{x_t^{h*}, \alpha_t^{h*}, s_t^{h*}, l_t^{h*}\}_{t=1}^{\infty}$ solves:

$$\max_{x,w,\alpha,s,l} \quad \sum_{t=1}^{\infty} \beta_h^t u_h(x_t)$$

$$\begin{aligned} \text{s.t.:} \quad & \text{(i)} \quad x \in C, \quad \liminf_t \frac{l_t}{\prod_{\tau=1}^t R_\tau^*} \geq 0 \quad \text{and,} \\ & \text{(ii)} \quad \text{For all t,} \quad p_t^* x_t + s_t + l_t \leq m_t, \\ & m_t = \sum_k \alpha_{k\,t-1} s_{t-1} r_{k\,t-1}^* + R_{t-1}^* l_{t-1}, \\ & \sum_k \alpha_{k\,t} = 1, \quad \alpha_t \geq 0, \quad s_t \geq 0. \end{aligned}$$

3. For all firms k,

$$(w_t^{k-}, w_{t+1}^{k+}) \in d^k(p_t^*, p_{t+1}^*, \sum_l \alpha_{kt}^h s_t^h).$$

4. At every date t,

$$\begin{aligned} & \sum_i x_t^{i*} + \sum_h x_t^{h*} + \sum_h w_t^{h*} - \sum_i e^i = 0, \\ & \sum_i l_t^i + \sum_h l_t^h = 0. \end{aligned}$$

where $\{w_1^{k*+}\}_{k=1}^K$ and $\{w_1^{hk*+}\}_{h=1,k=1}^{H\ \ K}$ are given such that each $w_1^{hk*+} \geq 0$ and $\sum_{h=1}^H w_1^{hk*+} = w_1^{k*+}$, and for each h, $m_1^h = p_1^{\mathbf{Con}*} \sum_{k=1}^K w_1^{hk*+}$.

Aside from the details of the loan markets, this definition differs from the previous constrained equilibrium definition in that here a firm is not owned by a single capitalist, and in that here the economy must be initialized by distributing ownership shares of pre-existing production among capitalists.

In a rational expectations constrained financial equilibrium only those firms that offer the maximal rate of return on investment will receive any funds. So no inefficient firms will ever operate if for each technology at least one firm with access to the technology maximizes constrained profit.

The following theorem shows that this system of markets — spot markets for consumption loans and financial capital — is dynamically complete if all consumers have rational expectations. If consumers have rational expectations, then all equilibrium allocations are Pareto optimal.

Theorem 6.1: Suppose Assumptions I, C, and D hold. Any rational expectations constrained financial equilibrium is Pareto optimal

6.1. Evolution and Optimality with Dynamically Incomplete Markets

In a rational expectations constrained financial equilibrium no selection over firms (other than the trivial and immediate selection at the beginning of time) occurs so this is not an appropriate structure in which to ask about selection for profit maximizing firms. However, with the financial markets described above inefficient firms may attract investment if some investors do not have rational expectations. In this case, the selection question shifts from direct selection over firms to the effect on firms of selection over investors with differing expectations. The interesting questions are: Will investors with rational expectations be selected for? Will this cause inefficient firms to eventually disappear? Will the economy converge to a rational expectations equilibrium?

The definition of rational expectations constrained financial equilibrium has rational expectations built into it but an extension to allow for differing expectations is easy. When a consumer makes his consumption, savings and investment plans he does so at each date using whatever expectations he has at that date about future prices. These expectations may be conditioned on any information that the consumer has. This information is all publicly available information, current and past prices, as well as the consumers own past choices and his current wealth. A worker's decision problem yields consumption and savings decisions at each date. A capitatlist's decision problem yields consumption, savings and investment choices as well a choice among the set of available production plans for his firm at each date. The market clearing conditions for inputs and outputs at each date are unchanged. We will refer to an equilibrium with financial markets when some consumers may not have rational expectations as a constrained financial equilibrium.

Whether or not rational expectations are selected for depends on what is meant by "rational expectations". There are (at least) two possible definitions. "Rational expectations" is a constraint on investors' beliefs. The first candidate constrains beliefs in equilibrium, but not outside equilibrium. These expectations can be viewed as either forecasting a particular price sequence or as using a forecasting rule mapping observable information into predicted prices, but in either case, there are no constraints on forecasts from data that are not generated in equilibrium. We call these expectations "narrow sense" rational. We say that capitalists have narrow sense rational expectations if there is a constrained financial equilibrium for some distribution of wealth that is a rational expectations constrained financial equilibrium. The second candidate is expectations that are correct both in and out of equilibrium. That is, expectations that always forecast correctly regardless of the behavior of other traders. We call such expectations "wide sense" rational.

Individuals with narrow sense rational expectations need not forecast prices correctly in an economy in which some individuals have incorrect expectations. Thus they may make inferior investments and their share of wealth need not converge to one. As a result the economy need not become even asymptotically efficient. We addressed a closely related question in Blume and Easley (1982) where we showed that rational expectations equilibria need not be locally stable under a simple learning dynamic.

Example 6.1: To see how selection can fail even with financial markets we add financial markets to the economy of Example 5.1. In that example there are two technologies each producing a mix of the two output goods from a single input. Any allocation of input to these two firms results in a point on the production possibility frontier. So to make production inefficiency possible we also add a third dominated technology. Technology three produces 0.8 times as much as does technology 2 from a unit of input. This economy has a unique rational expectations constrained financial equilibrium (RECFE) with constant input and output prices, $p_z = 1, p_x = 0.90009$ and $p_y = 0.9991$, and a constant gross rate of return on loans, $R_t = 1/\beta$. All consumers discount at rate β, so with constant goods prices and a gross rate of return on loans of $1/\beta$ the loan market does not operate. The constant outputs and the share of financial capital that is invested in firms 1 and 2 is a function of the utility parameter ρ. Firm 3 offers a lower rate of return than does firm 2 and so it never operates. We assume that $\rho = -3.0$. The RECFE share of financial capital that is invested in technology one is 0.533305 and the resulting outputs are $(x, y) = (0.5338, 0.5200)$.

Suppose that all workers, and capitalist one, always forecast the RECFE prices. Thus they have narrow sense rational expectations. Capitalist two is irrational. He believes that prices will be constant over time but he does not forecast the rational expectations prices. Exactly what prices he forecasts do not matter (because of the form of his utility function), all that matters is how he chooses to allocate his savings between the firms. We assume that he always invests share 0.875 of his wealth in technology one and the remainder in technology three. Because all consumers forecast constant prices, and discount at rate β, the loan market clears with no trade at a constant gross rate of return of $1/\beta$. Goods prices will vary with the wealth of the capitalists because of capitalist two's irrationality. The economy will be in a RECFE, and thus achieve a Pareto optimal allocation, only when capitalist one has all of the wealth. When the wealth of capitalist one is $1/\beta$ he must invest share 0.533305 of his wealth in technology one and the remainder in technology two in order to support a RECFE. At any other wealth level his expectations and allocation of wealth between the two efficient firms is not tied down by the narrow sense rational expectations hypothesis.

Because capitalist one always forecasts the RECFE prices he believes that the rate of return on investment in either efficient firm is $1/\beta$ and that the rate of return on investment in the inefficient firm is less than $1/\beta$ (as it is in a RECFE) so he is indifferent over investment shares between firms one and two. We assume that when he has wealth w he invests share $0.5(\sin(2\pi+0.0666594w/\beta)+1)$ of it technology one and the remainder in technology two. This rule has the property that when capitalist one has all of the wealth in the economy, $w = 1/\beta$, the share invested in technology one, 0.533305, supports the RECFE. Other than the price forecast and allocation at wealth share one the structure of this rule is not tied down by the narrow sense rationality hypothesis.[3]

If capitalist two has all of the wealth in the economy then the allocation of financial capital is incorrect and the equilibrium allocation is not a RECFE allocation. If capitalist one has all of the wealth then he invests correctly, the rational expectations prices are realized and the allocation is

---

[3] Only two properties of this rule matter for our results. First, at some wealth share less than one for the rational capitalist the two capitalists invest so as to have equal rates of return and at this wealth the slope of capitalist ones allocation rule is positive. Second, at wealth share one the rational capitalist invests so as to support the RECFE and at this point the slope of the rule is positive.

the RECFE allocation. The question is what happens if initially both capitalists have some wealth.

Figure 4 illustrates the map from the wealth of capitalist one at time t to his wealth at time $t + 1$ for an economy with $\beta = 0.9$. This equation of evolution has five steady states, but only two of them are locally stable. The steady state in which capitalist one has all of the wealth is one of these locally stable steady states. But its basin of attraction is tiny. Only if the initial wealth of capitalist one is at least 1.109 (a wealth share of 0.998) will his wealth share converge to one. To see why this is the case note that as capitalist one's wealth falls from $1/\beta$ he invests less than the RECFE fraction in technology one and correspondingly more than the RECFE fraction in technology two. Capitalist two invests fraction 0.875 (more than the RECFE fraction) of his small wealth in technology one and the rest in the dominated technology. The result is that less than the RECFE fraction of total wealth is invested in technology one and thus the rate of return on technology one is greater than on technology two. For wealth of capitalist one below 1.109 capitalist two has a greater rate of return on his investments than does capitalist one and so two's wealth share grows. Finally, as Figure 4 illustrates no other wealth levels are mapped into a wealth for capitalist one of 1.109 or more.

The other interesting locally stable steady state occurs at wealth 0.1639 for capitalist one. At this wealth for capitalist one the two capitalists have the same rate of return on investment–thus a steady state of the wealth dynamic. More important is the fact that this steady state is stable; the derivative of the wealth dynamic is -0.1222 at 0.1639. To see why note that if capitalist one has slightly more wealth than 0.1639 he will invest a bit more than 0.875 of it technology one. In aggregate, technology one is being allocated too much capital so the rate of return on investment in technology one, 0.539, is much lower than that on technology two, 5.282, or technology three, 4.226. Thus capitalist one will lose relative to capitalist two and his wealth will shrink back towards the steady state. □

FIG 4. HERE.

At the interior locally stable steady state in Example 6.1 the rates of return on the two efficient technologies differ by a factor of about ten. It may seem that the rational capitalist should notice this difference and thus begin to invest more in technology two. We could change his rule to incorporate this idea by requiring that when he has wealth of 0.1639 he invests only in technology 2. Of course if he does so then the rates of return and resulting optimal investment choice will change. Making simple modifications of this sort to the rational capitalist's investment rule will not solve the "problem"–that the evolution depends on expectations out of a rational expectations equilibrium. The following example provides an economy in which there is no obvious change in the narrow sense rational expectations that would make the capitalist better off.

Example 6.2: Suppose that firm 3 in Example 6.1 produces 0.001 units of good x and 0.05 units of good y from one unit of input. Everything else in that example is unchanged. Now the map from the wealth of the narrow sense rational capitalist at one date to his wealth at the next date is given by Figure 5.

FIG 5. HERE.

This economy has two interior steady states: one at $w = 0.6511$ and one at $w = 1.1025$. Neither are locally stable. The only locally stable steady state occurs when the rational capitalist has all of the wealth. The basin of attraction for this steady state is $[1.1025, \frac{1}{\beta}]$. Again unless the rational capitalist begins with nearly all of the wealth the economy will not converge to a REE.

Figure 6 provides plots of the time paths of the rational capitalist's wealth from two initial points: One in the basin of attraction of the REE and one just outside it. The dark band in the figure corresponds to the meta-stable region where the slope of the equation of evolution is nearly one. This region is not stable, but transitions through it are very slow.

In this economy unless the narrow sense rational capitalist begins with nearly all of the wealth, or none of it, prices are chaotic. Now it is far from obvious how the narrow sense rational capitalist should change his behavior in order to make better investments. □

FIG 6. HERE.

Alternatively we could require a rational capitalist to always invest optimally. To do so he would have to be able to predict rates of return when the economy is not in a RECFE. Thus he would have wide sense rational expectations. If we assume that rational capitalists have wide sense rational expectations then convergence to a CFE is assured. In fact, it is easy to show, using Euler equation arguments similar to those in the proofs of our theorems in Section 3 that investors with wide sense rational expectations will be selected for. More carefully, if there is at least one investor with wide sense rational expectations and a discount factor as large as any other discount factor in the economy then this investor will survive and prices will converge to rational expectations equilibrium prices. The following example shows how this occurs and why it is not interesting.

Example 6.3: Suppose there are four firms using two technologies. Each technology is employed by one profit maximizing firm, firms 1 and 3, and one non-maximizing firm, firms 2 and 4. Each technology produces one good, and both goods are desired by consumers. Suppose there are two capitalists with equal discount factors, one of whom has wide sense rational expectations. The rational investor will always invest in the profit maximizing technologies, consequently his share of total investment will grow relative to the investor investing in non-maximizing firms. It can be shown using Euler equation arguments that the share of investment belonging to the investor who invests in non-maximizing firms converges to 0. Thus, in this example, investors with “bad beliefs” are driven out.

But investors with incorrect beliefs who nonetheless always invest in profit maximizing firms need not be driven out. Suppose that capitalist one has wide sense rational expectations and that capitalist two knows the true rate of return to investment in firm 1, but underestimates all the others. Only firms 1 and 3 receive any investment funds. We can show that their profitability must be identical after some finite number of dates. Furthermore this happens as soon as investor one, the rational investor, is wealthy enough that making all his investment in firm 3 is enough to guarantee that its rate of return is less than firm 1.

But how is this maintained? Capitalist two invests all his money in firm 1, and capitalist one, the rational investor, allocates his money between firms 1 and 3 so as to guarantee equal rates of return. Suppose now far off in time, after this steady state is reached, capitalist two changes his investment rule so that in every 13th period he places all of his investment in firm 3. If capitalist one leaves his investment alone, firm 3 will earn less than firm 1, so investing in firm 3 would contradict the wide sense rational expectations hypothesis. If capitalist one invests everything he has in firm 1, firm 1 will earn a lower rate of return than firm 3, which is also inconsistent with wide sense rational expectations. Consequently, capitalist one must adjust his investment every thirteenth period so as to just offset capitalist two's behavior. □

The wide sense rational expectations hypothesis requires that "rational investment" be responsive to the investment of irrational actors. We believe that this kind of information requirement is inconsistent with the spirit of competitive analysis. To assume that in any economy there is at least one capitalist who always correctly forecasts endogenous prices begs the question of how a capitalist whose behavior is so carefully tuned to the structure of the economy, including the behavior of any irrational capitalists, could arise.

7. Conclusion

We have investigated the market selection hypothesis — the hypothesis that markets favor the survival and growth of profit maximizing firms — in a fairly special general equilibrium model. Nonetheless we believe that the lessons our analysis teaches hold quite generally.

First, there are market processes, such as our retained earning dynamic, that encourage the growth of more profitable firms at the expense of less profitable firms. If these processes are a dominant force in the economy, this would seem to justify the use of the profit maximizing hypothesis in equilibrium analysis as Friedman argued. However, other market forces may work against the selection of profit-maximizing firms.

Second, and more importantly, we show that this defense of the use of the profit maximization hypothesis leads to welfare conclusions at odds with the orthodox welfare analysis of competitive economies. Our model is constructed to insure that any stable steady state of the financial capital dynamic is a competitive equilibrium and is thus Pareto optimal. However, we find that equilibrium paths need not converge at all. In fact, we show through examples that they can exhibit complex dynamics. The resulting path of allocations is not in any sense optimal. Although our model is specific the forces responsible for this lack of convergence are not special. The key to our conclusion is that we study the evolution of financial capital in a full equilibrium model. It is of course true that at each date the firms that are most profitable grow the fastest. However, the is no reason for any firm to be uniformly more profitable than another firm unless the two firms use the same technology and one does not maximize profits (this generates the first conclusion). So as prices evolve, as they must because the allocation of financial capital is evolving, firms using technologies that are not on the production possibility frontier and which would therefore not be used in a competitive equilibrium can be more profitable than "efficient' firms. The resulting endogeniety of profitability can produce cyclic and even chaotic equilibria.

We view our analysis as showing that Koopmans' cautionary remarks about the use of natural selection as the basis for profit maximization are correct. We show that it is simply not appropriate to argue for profit maximization on the basis of natural selection and then replace natural selection by profit maximization in either static or dynamic equilibrium analysis. It may be that profit maximizing behavior is a useful hypothesis, but the usefulness of natural selection as a defense of profit maximization is very limited.

In this paper we have investigated selection for profit maximization and its use in general equilibrium analysis in an economy without stochastic shocks. Dutta and Radner (1996) demonstrate in an uncertain world that firm decision rules which maximize long run survival probabilities are not those which maximize expected profits. Studying market selection with uncertainty is particularly interesting because when profits are random, and the firm cannot be valued through arbitrage, it is unclear what objective to attribute to a firm. It is not obvious that capitalists would agree on expected profit maximization or on any other objective for the firm. In this case it is particularly interesting to see what behavioral rules the market selects for. We conjecture that, just as the investment market of Blume and Easley (1992) favors those rules with higher expected log returns, constrained equilibrium paths for an economy with uncertainty will favor those firm decision rules with higher expected log revenues.

8. Appendix

Proof of Lemma 2.1: Multiplying price vectors by positive scalars leaves the workers' budget sets unchanged, so their demand is invariant to the change in prices. The same is true for capitalists. To see this, consider a plan $(x_t, \omega_t)_{t\geq 1}$ in capitalist h's budget set. First observe that, for fixed ω_t^+, the set of affordable (x_t, ω_t^-) pairs is invariant to the proposed change in scale of prices. Finally observe that, due to homogeneity, $(\omega_t^-, \omega_{t+1}^+) \in d^h(p_t, p_{t+1}, p_t\omega_t^-)$ if and only if $(\omega_t^-, \omega_{t+1}^+) \in d^h(\lambda_t p_t, \lambda_{t+1} p_{t+1}, \lambda_t p_t \omega_t^+)$. $\square$

Proof of Theorem 3.1: Under our assumptions, each capitalist h and k's optimal path solves the optimization problem

$$\begin{aligned} \max \quad & \sum_t \beta^{t-1} v^j(p_t, z_t) \\ \text{s.t. for all } t \quad & z_t + y_t = m_t \\ & m_{t+1} = \rho^j(p_t^{j-}, p_{t+1}^{j+}, y_t) \end{aligned}$$

for $j = h, k$, where m_t is revenue at the beginning of period t, z_t is consumption expenditure, y_t is expenditure on inputs and $v(p, z)$ is the capitalist's one-period indirect utility function evaluated at prices p and expenditure on consumption z. The Euler equation is necessary because of Assumptions I and R. Therefore along any equilibrium path,

$$v_z^j(p_t, z_t) = \beta_j \rho_{yt}^j v_z^j(p_{t+1}, z_{t+1}).$$

Consequently,

$$\begin{aligned}\frac{v_z^h(p_{t+1}, z_{t+1}^h)}{v_z^k(p_{t+1}, z_{t+1}^k)} &= \frac{\beta_k \rho_{yt}^k}{\beta_h \rho_{yt}^h} \frac{v_z^h(p_t, z_t^h)}{v_z^k(p_t, z_t^k)} \\ &= \frac{v_z^h(p_1, z_1^h)}{v_z^k(p_1, z_1^k)} \prod_{\tau=1}^{t-1} \frac{\beta_k \rho_{y\tau}^k}{\beta_h \rho_{y\tau}^h}.\end{aligned}$$

From the definition of the marginal utility of income it follows that for any good consumption good j,

$$\frac{D_j u^h(c_{t+1}^h)}{D_j u^k(c_{t+1}^k)} = \frac{v_z^h(p_1, z_1^h)}{v_z^k(p_1, z_1^k)} \prod_{\tau=1}^{t-1} \frac{\beta_k \rho_{y\tau}^k}{\beta_h \rho_{y\tau}^h}. \qquad (*)$$

Suppose that $\prod_{\tau=1}^{t-1} \beta_k \rho_{y\tau}^k / \beta_h \rho_{y\tau}^h$ converges to 0. Then the left hand side of the final inequality converges to 0. Since consumption is bounded from above along any equilibrium path, the numerator of the right hand side is bounded away from 0. Consequently the denominator of the right hand side must be converging to $+\infty$, and so c_t^k converges to 0.

Finally, we need to show that capitalist k's share of input purchases goes to 0. Suppose not. Then there is an $\epsilon > 0$ such that infinitely often he can purchase at least fraction ϵ of the aggregate endowment. Since preferences are strictly monotone, in any such period he could use it to produce a consumption bundle that would give him utility $\delta > 0$ were he to consume it himself. If he carries out this plan at some date far in the future, its utility exceeds the continuation utility of the optimal plan with $c_t^k \to 0$. This is a contradiction. □

Proof of Corollary 3.1: Consider equation $(*)$ in the proof of Theorem 3.1. For the profit-maximizing firm h, $\rho_{yt}^h = r_t^h$, while for the other firm Assumption R implies that $\rho_{yt}^k \leq r_t^k$. Consequently

$$\frac{v_z^h(p_{t+1}, z_{t+1}^h)}{v_z^k(p_{t+1}, z_{t+1}^k)} \leq \frac{v_z^h(p_1, z_1^h)}{v_z^k(p_1, z_1^k)} \prod_{\tau=1}^{t-1} \frac{r_\tau^k}{r_\tau^h}$$

and the rest of the argument follows as in the proof of the Theorem. □

Proof of Corollary 3.2: If both firms are profit maximizers, then $\rho_{yt}^i = r_t^j$, and the result now follows immediately from Theorem 3.1. □

Proof of Theorem 4.1: In equilibrium, utility maximization for workers implies that $\sum_t q_t e^i < \infty$. Consequently the first welfare theorem is valid, and so every competitive equilibrium is Pareto optimal. Let $\left((\omega_t^h, x_t^h)_{h=1}^H, (x_t^i)_{i=1}^I\right)$ denote the equilibrium allocation, and consider the allocation with the same first period consumptions, and such that the following properties hold: For all $t \geq 1$, $\omega_t^{h-} = (1-\beta)\sum_{\tau\geq 1}\beta^{\tau-1}\omega_\tau^{h-}$. For all $t \geq 2$, $\omega_t^{h+} = (1-\beta)\sum_{\tau\geq 2}\beta^{\tau-2}\omega_\tau^{h+}$. For all $t \geq 2$ and for every consumer j (capitalist or worker), $x_t^j = (1-\beta)\sum_{\tau\geq 2}\beta^{\tau-2}x_\tau^j$. This allocation is feasible. If the equilibrium consumption plan is not stationary, this allocation is also Pareto preferred, which is a contradiction. □

Proof of Theorem 4.3: Because the consumption path is stationary, all the output prices are colinear. Rescale prices (according to Theorem 2.1) so that $p_t^{\mathbf{Con}} = \beta^{t-1}p_1^{\mathbf{Con}}$. This price sequence supports the stationary consumption path of all consumers in the competitive equilibrium consumer choice problem.

It follows from Theorem 3.1 that all active firms maximize constrained profits, but it remains to show that all active firms maximize (unconstrained) profits. Output price ratios are constant and the level is falling at rate β. Consequently the value of each capitalist's output falls at rate β. But, due to piecewise linearity, the capitalists problem does not restrict input prices. However, since the output prices are falling at rate β, it follows that the value of each consumer's consumption falls at the same rate. Since each worker's budget constraint is satisfied, the value of each worker's endowment falls at rate β. Consequently the value of aggregate expenditures falls at the same rate. Since each capitalist's share of input expenditure is constant, each capitalist's input expenditures falls at rate β.

We turn now to the decision problem of a typical capitalist. This capitalist is spending amount z_t on consumption and y_t on inputs in period t. Let $r = p_{t+1}\omega_{t+1}^+/p_t\omega_t^-$ denote the gross rate of return on a dollar invested in the firm at time t. (We have already seen that this number is constant through time). Constrained profit-maximization implies that each firm is run so as to maximize r. So we only need show that $r = 1$. The capitalist solves the following decision problem:

$$\begin{aligned}
\max_{\{y_t, z_t\}} \quad & \sum_{t=1}^{\infty} \beta^{t-1} v(\beta^{t-1}p_1^{\mathbf{Con}}, z_t) \\
\text{s.t. for all } t \quad & y_t + z_t = m_t, \\
& m_{t+1} = r y_t, \\
& m_1 > 0 \text{ given}, \\
& 0 \leq y_t \leq m_t,
\end{aligned}$$

where $v(p, z)$ is the capitalist's indirect utility function for the one-period problem. The Euler equation is

$$v_z(p_t, z_t) = \beta r v_z(p_{t+1}, z_{t+1}).$$

From the 0-degree homogeneity of indirect utility and stationarity,

$$v_z(p_t, z_t) = \beta r v_z(p_{t+1}, z_{t+1}) = r v_z(\beta^{-1}p_{t+1}, \beta^{-1}z_{t+1}) = r v_z(p_t, z_t).$$

Assumption I implies that $v_z > 0$, so $r = 1$. □

The proof of Theorem 4.4 requires a result about the continuity properties of constrained equilibrium.

Lemma 8.1: Suppose that Assumptions I, C and ND holds, and that $\left(p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H\right)_{t=1}^\infty$ is a constrained equilibrium. If the equilibrium allocation at date t converges to $\left((x^{i*})_{i=1}^I, (x^{h*}, \omega^{h*})_{h=1}^H\right)$ as t grows large, then there are positive scalars λ_t and a price vector p^* such that:

(i) $\left(\beta^{-(t-1)}p^*, (x^{i*})_{i=1}^I, (x^{h*}, \omega^{h*})_{h=1}^H\right)_{t=1}^\infty$ is a constrained equilibrium, and

(ii) $\beta^{-(t-1)}\lambda_t p_t$ converges to p^*.

Proof: Consider the consumption goods price sequence $\{\|p_t^{\mathbf{Con}}\|^{-1}p_t^{\mathbf{Con}}\}_{t=1}^\infty$. Then the consumption goods prices all lie in a compact set. Any sub-sequential limit supports each x_i^* and x_h^*. From our assumptions on preferences there is a unique such consumption goods price vector of length 1. Call it $p^{\mathbf{Con}*}$, and observe that the sequence of consumption goods prices $p_t^{\mathbf{Con}}$ converges to the ray defined by $p^{\mathbf{Con}*}$. Assumption ND implies there is a unique $p^{\mathbf{Inp}*}$ which solves the workers' budget constraints when consumption goods prices are $p^{\mathbf{Con}*}$ and consumptions are x^{i*}. Upper hemi-continuity of the solution correspondence for linear equations implies that $p^{\mathbf{Inp}*}$ is the limit of the sequence $\{\|p_t^{\mathbf{Con}}\|^{-1}p_t^{\mathbf{Inp}}\}_{t=1}^\infty$. Taking $\lambda_t = \beta^{t-1}\|p_t^{\mathbf{Con}}\|$ satisfies (ii). From Lemma 2.1, $\left(\beta^{-(t-1)}\lambda_t p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H\right)_{t=1}^\infty$ is a constrained equilibrium such that, in addition to the convergence of the allocation, prices converge to p^*. The properties of equilibrium are all closed, and so $\left(p^*, (x^{i*})_{i=1}^I, (x^{h*}, \omega^{h*})_{h=1}^H\right)_{t=1}^\infty$ is a constrained equilibrium. Finally, renormalizing prices as per Lemma 2.1 gives (i). □

Proof of Theorem 4.4: Let $\left(\beta^{(t-1)}p, (x^i)_{i=1}^I, (x^h, \omega^h)_{h=1}^H\right)_{t=1}^\infty$ be a locally stable constrained equilibrium, and let $\left(\beta^{(t-1)}p_t, (x_t^i)_{i=1}^I, (x_t^h, \omega_t^h)_{h=1}^H\right)_{t=1}^\infty$ denote a constrained equilibrium whose allocation converges to the stationary allocation. According to Lemma 8.1, there is no loss of generality in assuming that $\beta^{-(t-1)}p_t$ converges to p. We will refer to the stationary equilibrium and the converging equilibrium, respectively.

For each worker and capitalist, $\beta^{-(t-1)}p_t c_t^j$ is converging to a limit z^j, and for each capitalist, $\beta^{-(t-1)}p_t\omega_t^{h-}$ converges. The argument of Theorem 4.3's proof shows that all active firms are profit maximizing and earning 0 profits. It remains only to show that a vanishing firm could not make positive profits if it became active in the limit.

Assumption I implies that the Euler equation holds along any equilibrium path. Thus

$$v_m^h(p_t^{\mathbf{Con}}, z_t^h) = v_m^h(p_1^{\mathbf{Con}}, z_1)\left(\beta^{t-1}\prod_{\tau=1}^{t-1} r_\tau^h\right)^{-1}$$

where z_t is the expenditure on date t consumption goods and r_t^h is the gross rate of return on a dollar invested in firm h for one period at date t. Since indirect utility is homogeneous of degree

0, its partial derivatives are homogeneous of degree -1. Consequently for any capitalist h,

$$\lim_t v_m^h(\beta^{-(t-1)}p_t^{\mathbf{Con}}, \beta^{-(t-1)}z_t^h) = \lim_t \frac{1}{\prod_{\tau=1}^{t-1} r_\tau^h} v^m(p_1^{\mathbf{Con}}, z_1^h).$$

The price sequence $\beta^{-(t-1)}p_t^{\mathbf{Con}}$ converges to $p^{\mathbf{Con}}$. If firm h vanishes, then

$$0 \le \lim_t \beta^{-(t-1)}z_t^h \le \lim_t \beta^{-(t-1)}p_t^{\mathbf{Con}}\omega_t^{h+} = p^{\mathbf{Con}} \cdot 0 = 0$$

Assumption I implies that the left hand limit is $+\infty$ for vanishing firms. Therefore $\lim_t \prod_{\tau=1}^{t} r_t^h = 0$, that is, the long run gross rate of return on investment in firm h is 0.

Finally, observe that $r_t^h = p_{t+1}^{\mathbf{Con}}\omega_{t+1}^{h+}/p_t^{\mathbf{Inp}}\omega_t^{h-}$ converges to $r^h = \beta p^{\mathbf{Con}}\omega^{h+}/p^{\mathbf{Inp}}w^{h-}$, the rate of return for firm h in the stationary equilibrium. If firm h is inactive in the stationary equilibrium, it is vanishing in the converging equilibrium. But if $r^h > 1$, then for t large enough, $r_t^h > 1$, which contradicts the conclusion of the previous paragraph, that the long run gross rate of return on firm h investment is 0. □

Proof of Theorem 6.1: We show that in a rational expectations CFE markets are dynamically complete and all operating firms maximize (unconstrained) profits. Thus the equilibrium allocation is a complete markets competitive equilibrium allocation (Def. 2.3) which is Pareto optimal by the first welfare theorem. By Assumption D there is, for each technology, at least one firm that maximizes profit. All capitalist have rational expectations so clearly no non-maximizing firm will receive any investment. We thus ignore such firms.

Lemma 8.2: $r_t^* = R_t^*$.

Proof: Suppose at some date t, $r_t^* > R_t^*$. Let $(x_t^{h*}, \alpha_t^{h*}, s_t^{h*}, l_t^{h*})_{t=1}^\infty$ be the optimal plan for capitalist h. Consider the plan $(\hat{x}_t^h, \alpha_t^{h*}, \hat{s}_t^h, \hat{l}_t^h)_{t=1}^\infty$ which agrees with the supposed optimal plan at all dates other than t and $t+1$ and has: $\hat{l}_t^h = l_t^{h*} - 1$, $\hat{s}_t^h = s_t^{h*} + 1$, $\hat{x}_t^h = x_t^{h*}$, $\hat{l}_{t+1}^h = l_{t+1}^{h*}$, $\hat{s}_{t+1}^h = s_{t+1}^{h*}$, and $\hat{x}_{j\,t+1}^h = x_{j\,t+1}^{*h} + (r_t^* - R_t^*)/p_{j\,t+1}^*$ for each good j. This plan is clearly feasible and has a higher value than the supposed optimal plan. So $r_t^* \le R_t^*$.

Now suppose that at some t, $r_t^* < R_t^*$. Then clearly $s_t^{h*} = 0$ for all capitalists h. Then $\omega_{t+1}^{k*+} = 0$ for all firms k. Then by Assumption B, $\| p_{t+1}^{C*} \| = \infty$. So $r_t^* \ge R_t^*$. □

Definition 8.1: The present value of profits from $(\omega_{t-1}^{k-}, \omega_t^{k+})$ is

$$\pi_t^k = \frac{p_t^{C*}\omega_t^{k+}}{R_t^*} - p_{t-1}^{I*}\omega_{t-1}^{k-}.$$

Lemma 8.3: $\pi_t^k = 0$ for all k, t.

Proof: For any inactive firm profits are clearly 0. Suppose that firm k is active. Then from Lemma 8.2 and the observation that all active firms earn the same rate of return,

$$R_t^* = r_t^* = \frac{p_t^{C*}\omega_t^{k*+}}{p_{t-1}^{I*}\omega_{t-1}^{k*-}}.$$

So,

$$\pi_t^{k*} = \frac{p_t^{C*}\omega_t^{k*+}}{R_t^*} - p_{t-1}^{I*}\omega_{t-1}^{k*-} = 0.$$

□

So in a CFE with rational expectations, all constrained profit maximizing firms are unconstrained profit maximizers.

Workers i's CFE budget set can be written as:

$$B^i(p^*, R^*) = \{x^i \in C : \text{there exists } l^i \text{ such that } p_t^* x_t^i + l_t^i \leq m_t^i = p_t^* e^i + R_t^* l_{t-1}^i \text{ for all } t,$$
$$l_0 = 0, \text{ and } \liminf_t \frac{l_t}{\prod_{\tau=1}^t R_t^*} \geq 0\}.$$

Definition 8.2: The complete markets budget set for worker i is:

$$\hat{B}^i(p^*, R^*) = \{x^i \in C : \liminf_T \sum_{t=1}^T \big(\frac{1}{\prod_{\tau=1}^t R_\tau^*}\big) p_t^* \cdot (e^i - x_t^i) \geq 0\}.$$

Lemma 8.4: $B^i(p^*, R^*) = \hat{B}^i(p^*, R^*)$ for all workers i.

Proof: (1) $B \subset \hat{B}$. Let $x^i \in \hat{B}^i(p^*, R^*)$. Define $z_t^i = p_t^* \cdot (e^i - x_t^i)$. Then

$$\liminf_T \sum_{t=1}^T \frac{z_t}{\prod_{\tau=1}^t R_t^*} \geq 0.$$

Define $l_0^i = 0$ and $l_t^i = z_t^i + R_t^* l_{t-1}^i$ for all $t \geq 1$. Note that for any T,

$$\frac{l_T^i}{\prod_{\tau=1}^T R_t^*} = \sum_{t=1}^T \frac{1}{\prod_{\tau=1}^t R_t^*} z_t^i.$$

Thus $x^i \in B^i(p^*, R^*)$ and is supported by the consumption loan sequence l^i.

(2)$\hat{B} \subset B$. For any $x^i \in B$ there is a consumption loan sequence l^i satisfying the constraints in B. Substitution shows that for any T,

$$\frac{l^i_T}{\prod_{\tau=1}^T R^*_t} = \sum_{t=1}^{T} (\frac{1}{\prod_{\tau=1}^t R^*_\tau}) p^*_t \cdot (e^i - x^i_t).$$

Thus $x^i \in \hat{B}$. □

By the observation that all active firms in period t earn rate of return r^*_t and by Lemma 8.2 the CFE budget set for capitalist h with rational expectations can be written as:

$$B^h(p^*, R^*) = \{x^h \in C : \text{there exists } l^h, s^h \text{ such that } p^*_t x^h_t + s^h_t + l^h_t \leq m^h_t = r^*_t s^h_{t-1} + R^*_t l^h_{t-1} \text{ with } s^h_t \geq 0$$

$$\text{for all } t \geq 1, m^h_1 \text{ given , and } \liminf_t \frac{l^h_t}{\prod_{\tau=1}^t R^*_t} \geq 0\}.$$

Definition 8.3: The complete markets budget set for capitalist h is

$$\hat{B}^h(p^*, R^*) = \{x^h \in C : \liminf_T \sum_{t=1}^{T} (\frac{1}{\prod_{\tau=1}^t R^*_\tau}) p^*_t \cdot x^h_t \leq m^h_i\}.$$

Lemma 8.5: $B^h(p^*, R^*) = \hat{B}^h(p^*, R^*)$ for all capitalists h.

Proof: See the proof of Lemma 8.4. □

As each consumer faces the competitive budget set, and each firm maximizes unconstrained profits, any CFE with rational expectations is a competitive equilibrium. In equilibrium,

$$\sum_{t=1}^{\infty} (\frac{1}{\prod_{t=1}^t R^*_t}) p^*_t e^i < \infty$$

for all i. So the competitive equilibrium is Pareto optimal. □

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