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On the Physical Reality of Wave-Particle Duality

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Existing experiments do not disprove local realistic quantum theories.

To overcome logical difficulties of the standard quantum theory, a non-deterministic irreversible local realistic theory was proposed, in which the wave-particle duality is objective in contrast to the complementarity principle. The statistical characteristics of these two theories can be distinguished by a proposed experiment within presently available technology.

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I. INTRODUCTION

One fundamental scientific principle is the belief of an external world describable by objective causal dynamical laws independent of human consciousness while susceptible to human observation and understanding, including the possibility of explaining living beings and mental activities. Although it is impossible to establish this belief empirically without adopting itself in the first place, it is certainly beneficial if a rational understanding of the world happens to be possible, and harmless otherwise. This principle has been seriously challenged by the standard quantum theory and its generally accepted Copenhagen interpretation, which claims that no objective (ie. independent of the observer) description of the physical reality is possible. Naturally we should not indorse such a drastic departure from scientific tradition without possessing overwhelming empirical evidence and exhausting possible objective explanations.

In two earlier letters [1,2], it was shown that the standard quantum theory encounters serious logical difficulties when combined with statistical physics or dealing with measurements, while the perceived difficulties of objective quantum theories are largely illusive. An alternative quantum theory was proposed in which matter propagates as waves and interacts as particles; The choice is independent of observation. It is called “mixed theory” (M) because both unitary evolution and irreversible random discrete jumps coexist objectively, in contrast to the standard theory as a “unitary theory” (U) with subjective wave-particle duality governed by Bohr’s complementarity principle [3]. In this letter we examine existing arguments and experiments concerning the viability of these theories, and propose a new experiment which could completely settle this issue using existing experimental technology.

We need to fix several concepts here. The term “causality” describes the existence of a conditional distribution $P(B|A)$ independent of the actual $P(A)$. It is generally irreversible [4]. The term “determinism” describes a nonrandom mapping from the past to the

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future. It can be either reversible or irreversible. The term “realism” describes the existence of a joint distribution of all the variables concerned. It does not imply determinism. The term “locality” is used as in relativity theory. For example, the Brownian motion is realistic, causal, irreversible and local, but not deterministic. Our new theory is in the same category. Generally there does not exist a causal reversible and nondeterministic reality.

II. THE EPR PARADOX AND WAVE-PARTICLE DUALITY

The idea that quantum phenomena generally do not allow objective description is best illustrated by the EPR paradox [5]. Under certain conditions a pair of identical particles are generated from a “zero state” and fly away in opposite directions. Their momentum $p_i$ and position $q_i$, $i = 1, 2$, may be measured later. It is believed that theory (U) predicts

(A1) The pair $(p_1, p_2)$ always reduces to eigenstates simultaneously.

(A2) The pair $(p_1, q_2)$ can be measured to infinite precision.

(A3) The pair $(p_1, p_2)$ can be measured to infinite precision, with $p_1 + p_2 = 0$.

(A4) There is an uncertainty limit for the measurement of $(p_i, q_i)$.

Now if we make measurement (A2) and use (A3) to infer the value of $p_2$, the whole experiment then determines $(p_2, q_2)$ to infinite precision, in violation of (A4). The standard explanation is that $p_2$ is not real in this situation as it is not measured. More specifically, there can be no valid prediction based on joint probability distribution of all the observables; The only one available concerns all the actually observed entities. Clearly this is in conflict with (A1), which is but an example of a peculiar yet essential feature of (U): the simultaneous reduction to eigenstates of all subsystems, even if spatially separated, when only a single subsystem is measured. This in itself already contains the seed of non-local interaction; It has to be renounced if any local theory (whether deterministic or not) is to be viable [6].

In theory (M) the two particles are replaced by two waves $\psi_1$ and $\psi_2$. The observables $p_i$ are not properties of the traveling waves alone, but are stochastically and independently determined when $\psi_i$ interacts with the detectors. The EPR paradox is resolved by the fact that measurements (A2) and (A3) require different arrangements of interaction with $\psi_2$; their simultaneous realization is limited by (A4). There is no “entanglement” with $\psi_1$. The conservation laws are satisfied by $\psi_1$ and $\psi_2$ when they depart, but otherwise impose no constraint on $p_1 + p_2$, because the latter also depends on the interaction with instrument during which $\psi_i$’s do not form an isolated system. By logical consistency we accept the local random quantum jumps as ingredients of any particle interactions, even those unrelated to any intention of measurement. If this is correct, i.e. if the EPR paradox can be resolved by simply abandoning (A1), then Einstein’s misgivings about the subjectivity of (U) would be vindicated, even though the solution is not a deterministic theory as hoped.

One difficulty in accepting (M) stems from the notion, due to Einstein [7], that photons can be regarded as particles not only when they interact but also when they propagate. Starting from energy conservation and entropy maximization, he obtained $ds_{\lambda\nu}/du_{\lambda\nu} = \beta$, where
where \( s_\nu \) and \( u_\nu \) are spectral densities of entropy and energy, and \( \beta = 1/kT \). For the high-frequency end of spectrum where Wien’s formula is applicable, he obtained

\[
s_\nu = \int \beta du_\nu = \int \frac{1}{\varepsilon} \log \left( \frac{u_\nu}{\varepsilon} \right) du_\nu = \frac{u_\nu}{\varepsilon} \left( 1 - \log \frac{u_\nu}{g_\nu} \right).
\]

At equilibrium, entropy \( s_\nu d\nu \) and energy \( u_\nu d\nu \) in any spectral interval \( d\nu \) are constant under adiabatic change of volume \( V_1 \rightarrow V_2 \), but the spectral density of eigenstates per volume changes as \( g_\nu \rightarrow g_\nu V_1/V_2 \). Therefore the total entropy changes by

\[
\Delta S = \frac{E}{\varepsilon} \log \frac{V_2}{V_1} = \log \left( \frac{V_2}{V_1} \right)^n,
\]

where \( n = E/\varepsilon \) is the number of energy quanta, distributed in the volume independently of each other. This conclusion is of course an artifact of Wien’s formula derived from Maxwell’s gas theory. If Rayleigh’s formula derived from Maxwell’s electromagnetic wave theory and equipartition assumption were used, entropy would be consistent with equipartition of energy among all eigenstates. In reality Planck’s formula is true so that entropy of thermal radiation is consistent neither with the equilibrium of pure particles nor pure waves, but with the Bose-Einstein distribution compatible with objective wave-particle duality [8,2]. Although this essential point was revealed soon afterwards [9-11], it seems to have been lost in the later developments and Einstein’s “heuristic viewpoint” of particles traveling in space became entrenched as if a proven fact.

### III. EXISTING EXPERIMENTS

As physical theories, the validity of theories (U) and (M) should be determined by experiments. (Ignore the internal inconsistency of (U) for the moment.) For practical reasons existing experiments are of EPRB-type suggested by Bohm [12,13], using spin or polarization in place of position and momentum. Under certain conditions a pair of photons departing in opposite directions are generated from a “zero state”, characterized by polarizations \( \theta \) and \( \theta \pm \pi \) uniformly distributed in \([0, 2\pi)\). (Polarization modulo \( \pi \) is insubstantial.) Two polarizers are placed in opposite sides of the source. The two light beams in their directions are split by the polarizers into four beams with polarizations \( \theta_i, i = 1, 2, 3, 4 \), where

\[
\theta_{1,2,3,4} = \alpha, \beta, \alpha + \frac{\pi}{2}, \beta + \frac{\pi}{2},
\]

and detected by detector \( i \) with efficiency \( \varepsilon \in (0, 1) \) (See Figure 1); The inefficiencies are assumed to be caused by independent random processes.

It is generally accepted that the source emission is a Poisson process; The emission number \( n \) in the direction of the polarizers in a given time window has a Poisson distribution with mean \( \lambda \), proportional to intensity and duration,

\[
\langle n \rangle = \lambda, \quad \langle n, n \rangle = \lambda, \quad \langle n^2 \rangle = \lambda + \lambda^2,
\]

where \( \langle \cdot \rangle \) and \( \langle \cdot, \cdot \rangle \) denote mean and covariance, respectively. The probability \( p_i \) of a detection at \( i \) per emission, with or without conditioning on \( \theta \), are

\[
p_i = P(n_i = 1|n = 1, \theta) = \varepsilon \cos^2(\theta_i - \theta),
\]
\[ \langle p_i \rangle = P(n_i = 1|n = 1) = \frac{\varepsilon}{2\pi} \int_0^{2\pi} \cos^2(\theta_i - \theta) d\theta = \frac{\varepsilon}{2}. \] (6)

\( \langle p_i \rangle \) is independent of \( \theta_i \). The detection number \( n_i \) at \( i \) in the given time interval has a Poisson distribution with mean
\[ \langle n_i \rangle = \langle p_i n \rangle = \langle p_i \rangle \langle n \rangle = \frac{\varepsilon \lambda}{2}. \] (7)

On the other hand, the probability \( p_{ij} = P(n_1 = 1, n_j = 1|n = 1, \theta) \) of joint detection at \( i \) and \( j \) per emission depends on \( \phi = \alpha - \beta \) and on the particular theory. It is generally believed that (U) predicts:

(U1) When \( \phi = 0 \) and conditional on emission, \( (n_1, n_2) \) is perfectly correlated, \( (n_1, n_3) \) is perfectly anti-correlated,
\[ p_{12} = \varepsilon p_1, \quad p_{13} = 0. \] (8)

(U2) Averaged over all \( \theta \),
\[ \langle p_{12} \rangle = \frac{\varepsilon^2}{4} (1 + \cos 2\phi) \in \frac{\varepsilon^2}{2} [0, 1]. \] (9)

In contrast, (M) predicts:

(M1) For \( i \neq j \) and conditional on emission, \( (n_i, n_j) \) is independent,
\[ p_{ij} = p_i p_j. \] (10)

(M2) Averaged over all \( \theta \),
\[ \langle p_{12} \rangle = \frac{\varepsilon^2}{4} (1 + \frac{1}{2} \cos 2\phi) \in \frac{\varepsilon^2}{2} [\frac{1}{4}, \frac{3}{4}]. \] (11)

These two theories can be distinguished experimentally by determining the coefficient \( c \) in the more general equation
\[ \langle p_{12} \rangle = \frac{\varepsilon^2}{4} (1 + c \cos 2\phi) \in \frac{\varepsilon^2}{2} \left[ \frac{1 - c}{2}, \frac{1 + c}{2} \right]. \] (12)

Most existing experiments attempt to determine quantities such as
\[ R = \frac{\langle p_{12} + p_{34} - p_{14} - p_{23} \rangle}{\langle p_{12} + p_{34} + p_{14} + p_{23} \rangle} = c \cos 2\phi, \] (13)
which are independent of \( \varepsilon \). It was shown that generally any local realistic theory satisfies Bell’s inequality (Bell [6] originally considered spins of spin-1/2 particles instead of photon polarizations and the inequality was in an equivalent but more complicated form.)
\[ \max |R| = c < 1. \] (14)

There are numerous claims in the literature that Bell’s inequality was violated in experiments [14–16]. It would seem unlikely that all these experiments could be flawed in the same way, but what really is at issue is how the data are analyzed. In these experiments the coincidences are averaged over a time interval instead of individually recorded, causing accidental coincidence. However, what is regarded as “accidental” is exactly what distinguishes the two theories. These experiments founder on this point, as they subtract a perceived “accidental coincidence” \( r \), compatible with (U1) but not with (M1), from \( \langle p_{ij} \rangle \) [17]. This effectively subtracts \( 4r \) from the denominator of \( R \) while leaving the numerator intact, boosting \( c \) up to unity whatever its true value is. Nevertheless these experiments do confirm the general relation (12) accurately, without committing to a particular value of \( c \).
IV. PROPOSAL FOR A NEW EXPERIMENT

To avoid similar hidden assumptions in data processing, our proposed new experiment uses direct predictions of

\[ Q_{ij}(\phi) = \langle n_i n_j \rangle / \langle n_i \rangle \]  

for actual \( \lambda > 0 \) so that the theories can be distinguished by raw data.

According to (M),

\[ p_{ij} = p_ip_j, \quad \langle n_i n_j \rangle = \langle p_ip_j \rangle \langle n^2 \rangle, \]  

\[ \langle p_{12} \rangle = \frac{\varepsilon^2}{2\pi} \int_0^{2\pi} \cos^2(\alpha - \theta) \cos^2(\beta - \theta) d\theta = \frac{\varepsilon^2}{4}(1 + \frac{1}{2} \cos 2\phi), \]  

\[ \langle p_{13} \rangle = \frac{\varepsilon^2}{2\pi} \int_0^{2\pi} \cos^2(\alpha - \theta) \sin^2(\alpha - \theta) d\theta = \frac{\varepsilon^2}{8}, \]  

\[ \langle p_{14} \rangle = \frac{\varepsilon^2}{2\pi} \int_0^{2\pi} \cos^2(\alpha - \theta) \sin^2(\beta - \theta) d\theta = \frac{\varepsilon^2}{4}(1 - \frac{1}{2} \cos 2\phi). \]  

Therefore,

\[ Q_{12}(\phi) = Q_{34}(\phi) = Q_{14}(\phi + \frac{\pi}{2}) = Q_{23}(\phi + \frac{\pi}{2}) = \frac{\varepsilon}{2}(1 + \frac{1}{2} \cos 2\phi)(1 + \lambda), \]  

\[ \max_{\phi} Q_{ij}(\phi) = Q_{12}(0) = Q_{34}(0) = Q_{14} \left(\frac{\pi}{2}\right) = Q_{23} \left(\frac{\pi}{2}\right) = \frac{3\varepsilon}{4}(1 + \lambda), \]  

\[ \min_{\phi} Q_{ij}(\phi) = Q_{12} \left(\frac{\pi}{2}\right) = Q_{34} \left(\frac{\pi}{2}\right) = Q_{14}(0) = Q_{23}(0) = \frac{\varepsilon}{4}(1 + \lambda). \]  

They add up consistently,

\[ \langle (n_1 + n_3)(n_2 + n_4) \rangle = \varepsilon^2 \langle n^2 \rangle. \]  

The predicted maximum and minimum of \( Q_{ij} \) versus \( \lambda \) are plotted in Figure 2.

According to (U), for \( \phi = 0 \), the pairs \( (n_1, n_2) \) and \( (n_3, n_4) \) are each totally correlated, while \( (n_1, n_3) \), \( (n_1, n_4) \), \( (n_2, n_3) \), \( (n_2, n_4) \) are each totally anti-correlated,

\[ p_{12} = \varepsilon p_1, \]  

\[ \langle p_{12} \rangle = \frac{\varepsilon^2}{2\pi} \int_0^{2\pi} \cos(\alpha - \theta) d\theta = \frac{\varepsilon^2}{2}, \]  

\[ \langle n_1 n_2 \rangle = \langle p_{12} \rangle \langle n^2 \rangle = \frac{\varepsilon^2}{2}(\lambda + \lambda^2), \]  

\[ \langle n_1 n_3 \rangle = \langle (n_1) \theta (n_3) \theta \rangle = \langle p_{13} \rangle \langle n^2 \rangle = \frac{\varepsilon^2}{8} \lambda^2. \]  

Therefore,

\[ \max_{\phi} Q_{ij}(\phi) = Q_{12}(0) = Q_{34}(0) = Q_{14} \left(\frac{\pi}{2}\right) = Q_{23} \left(\frac{\pi}{2}\right) = \varepsilon (1 + \lambda), \]  

\[ \min_{\phi} Q_{ij}(\phi) = Q_{12} \left(\frac{\pi}{2}\right) = Q_{34} \left(\frac{\pi}{2}\right) = Q_{14}(0) = Q_{23}(0) = Q_{13}(\phi) = Q_{24}(\phi) = \frac{\varepsilon}{4} \lambda. \]
These are inconsistent because
\[
\langle (n_1 + n_3)(n_2 + n_4) \rangle = \varepsilon^2 \left( \langle n^2 \rangle + \frac{1}{4} \langle n \rangle^2 \right). \tag{30}
\]

Due to such inconsistencies it is difficult to determine the prediction of (U) for arbitrary \( \phi \), but as (12) is verified by existing experiments, presumably the most acceptable prediction is a cosine curve bounded by \( \max Q \) and \( \min Q \), shown in Figure 3.

The differences in the predictions are sufficient to distinguish these two theories. On either graph there are two straight lines, determining four parameters. Two parameters are essentially scaling factors \( \lambda \) and \( \varepsilon \). The other two are dimensionless and distinguish the two theories:

(M):
\[
\frac{\min Q|_{\lambda=0}}{\max Q|_{\lambda=0}} = \frac{\partial_\lambda \min Q}{\partial_\lambda \max Q} = \frac{1}{3},
\tag{31}
\]

(U):
\[
\frac{\min Q|_{\lambda=0}}{\max Q|_{\lambda=0}} = 0, \quad \frac{\partial_\lambda \min Q}{\partial_\lambda \max Q} = \frac{1}{4}.
\tag{32}
\]

The deficiency of previous experiments now becomes obvious: They were conducted at a particular \( \lambda \) but compared with predictions at \( \lambda = 0 \) based on some \textit{ad hoc} notion of “accidental coincidence”.

The scaling factors \( \lambda \) and \( \varepsilon \) can also be determined, using \( \max Q = 0 \iff \lambda = -1 \), and \( \langle n_i \rangle = (\varepsilon/2)\lambda \). Furthermore, in (M) the value of \( Q_{12} \) does not depend on whether detectors 3 and 4 are working, because it can be obtained by marginalizing a joint distribution. In contrast, (U) does not admit such a joint distribution so that the result of any detector is dependent on all the other detectors.

As a consequence of the precise predictions of the zero point of \( \min Q \), both theories have zero \textit{a priori} probability of being correct. If experiments verify one of them with high precision, the other is quite confidently refuted. However, it is not inconceivable for experimental results to differ from both predictions. The most likely cause would be violation of some of the randomness and independence assumptions. For example, if the time window size is smaller or comparable to the photon frequency additional temporal dependencies is expected to occur. In practice \( \langle n_i n_j \rangle \) are usually recorded using an exponentially decaying window instead of a rectangular window, requiring modifications such as from \( \lambda^2 / 2 \) to \( \lambda + \lambda^2 / 2 \). The details depend on the actual set-up, but they are unlikely to blur the essential distinction between (U) and (M): whether the zero point of \( \min Q \) is at \( \lambda = 0 \) or coincides with that of \( \max Q \).

Incidently, this analysis also shows that such experimental tests rely on a large number of assumptions concerning the theories to be tested. Although they may distinguish two concrete theories sharply, they are unlikely to rule out a whole class of theories as often claimed in the literature.

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FIG. 1. Experimental set-up

FIG. 2. Prediction of the mixed theory

FIG. 3. Prediction of the unitary theory