Extremist Funding, Centrist Voters, and Candidate Divergence

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In an electoral framework of unidimensional two-candidate spatial competition with probabilistic voting, special interest groups present candidates with schedules that give the level of campaign contribution they will make for each feasible candidate policy location. Candidates, motivated by the desire both to hold office and to see their most preferred policy position enacted, spend contributions in an effort to “convince” voters of the merits of their announced position. In equilibrium, candidates adopt policy positions which balance the centrifugal force generated by special interests against the centripetal force to converge to the position of the expected median voter. Analytical and numerical methods are used to characterize candidate reaction functions and equilibrium locational outcomes. Even purely office-oriented candidates may diverge and policy-oriented candidates may adopt positions more extreme than their own ideal points. Where any particular candidate locates depends on the interaction of several factors including the candidate’s ideal policy, the relative utility she derives from policy realizations versus office-holding per se, her particular opponent’s location, the nature of the special-interest contribution schedules, and the technology by which campaign spending affects voters’ beliefs.

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1 Introduction

That two office-seeking candidates in a unidimensional world of voters with single-peaked preferences will converge to the ideal location of the median voter serves as the starting point for nearly all modern theoretical work on spatial voting. To be sure, numerous modifications of Downs (1957) and Black’s (1958) application of spatial competitive theory to the realm of politics have tempered any complete convergence results. But as Calvert (1985) has shown, deviations from complete convergence are “small” in the sense that the convergence result is not knife-edged. Rather, the convergence result is relaxed continuously as its underlying assumptions are so relaxed. Thus Calvert concludes that “convergence or near convergence is truly a basic property of electoral competition”, that any electoral system has a built-in pressure favoring convergence of competing alternatives toward the center of the voter distribution.

But actual electoral competition also includes numerous built-in pressures favoring divergence. A first possible source is given by Downs himself who admits “the possibility that parties will be kept from converging ideologically in a two-party system [by] the refusal of extremist voters to support either party if both become alike.” (p. 118) As Downs elaborates, in an intertemporal framework, extremist voters may rationally withhold their support from centrist parties thereby introducing a force towards divergence. Similarly, the need to appeal to extremist voters in the first stage of a two-stage election process may serve as a force towards candidate divergence. (See Aranson and Ordeshook, 1972). Moving beyond pure electoral considerations, policy-motivated candidates may diverge in order to achieve a more favorable post-election compromise with an opposition party. (See Alesina and Rosenthal, 1994, 1995, Chapter 5).

In this paper, I focus on still another source of divergence: policy-contingent donations made by extremist special-interest groups. In modern elections, special-interest groups can play a decisive role: they make donations to help fund campaigns; they proffer activists to help staff these; they make endorsements helping to deliver large blocks of votes; and they often campaign in parallel to the candidates helping to sway voter choices.\(^1\) It seems reasonable, then, to argue that special-interests disproportionately influence candidates relative

\(^1\)In the 1996 election cycle, it is estimated that special-interest groups will spend $2.5 billion on issue-oriented advertisements paralleling the U.S. Presidential and congressional races. A particularly salient historical example is the Willie Horton commercial aimed at the 1988 Dukakis presidential campaign which was produced and paid by the independent National Security Political Action Committee. See *New York Times* (1996, 1988).
to the actual number of voters of which they are composed. Almost by definition special-interest group preferences differ from those of the overall population, and to the extent that special-interest ideal positions are clustered at the extremes of spatial policy spaces, a built-in pressure towards candidate divergence may be introduced. Using the terminology of Cox (1990), electoral competition would then be characterized by both centripetal and centrifugal incentives.²

Introducing special-interest groups into electoral competition raises two key questions: what motivates the special interests to make campaign contributions? And what do the candidates do with the money? Large literatures in both economics and political science address the first question of special-interest motivation.³ In general, most models assume either that special interests seek to obtain private, non-policy constituent services – for instance, help in expediting matters through the federal bureaucracy, (Baron, 1989) – or that they hope to effect non-private policy goals. In the latter case, we can distinguish between two subsidiary contribution motives: “ideological”/“electoral” motives and “quid pro quo”/“influence” motives. (Welch, 1974; Grossman and Helpman, 1996). Electoral motives capture the idea that for fixed candidate policy positions, special interests may donate funds to help increase the election chances of their preferred candidate; influence motives, that special-interest contributions often seek to influence the candidates’ policy positions.

On the second question regarding what candidates do with the money, one class of models (Austen-Smith, 1987; Hinich and Munger, 1989, 1992, 1994; Gersbach, 1995) posit that risk-averse voters have only a noisy signal of candidates’ true policy locations. Candidates use campaign funds to decrease the variance associated with voters’ perceptions of their own policy positions and to increase the variance of voters’ perceptions of their opponents’ policy positions.⁴ While such an approach has much theoretical appeal, it seems doubtful that it captures what is actually going on in the real world. As Downs (1957) observes, candidates tend to “becloud their policies in a fog of ambiguity.” (p. 136) Similarly, Page (1976) argues,

Often candidates do not give any emphatic impression of their stands, whether certain or probabilistic; their chief endeavor seems to be to come as close as

² Cox (1990) relies on the interaction of multiparty competition and strategic voting to generate a centrifugal force.
³ Welch (1974) is an early reference and Morton and Cameron (1992) provide a partial survey.
⁴ Shepsle (1972) assumes a similar framework as these authors except that he argues for certain issues, voters are actually risk-loving so that candidates have an incentive to increase the noise associated with their true position.
possible to taking no stand. Except when pressed (in televised interviews or press conference question periods) they generally avoid specific issues altogether, and talk about such matters as the need for new leadership, the desirability of peace and prosperity, and the incompetence or wickedness of their opponents. The most specific stands are taken in obscure forums, where special audiences demand them but where they are easily missed by the general public.

An assumed use of campaign funds more consistent with such observations relies on the partitioning of the electorate into “informed” and “uninformed” voters. (Baron, 1994; Grossman and Helpman, 1996). Informed voters vote for candidates whose policy positions most closely resemble their own while uninformed voters’ choose solely according to campaign spending. In this framework, the need to raise campaign funds may cause candidates to take positions that undermine their support with informed voters. Hence, a tradeoff may exist between platforms that serve the “general interest”, attracting voters from the portion of the electorate that is well informed, versus platforms that garner special-interest contributions which are used to buy the support of uninformed voters. Similar in spirit are models in which campaign spending is used to “convince” voters that candidates’ policies map to favorable outcomes. (Congleton, 1989). Here the emphasis is on purely office-seeking candidates’ differentiating themselves from their opponents in order to win campaign funds which in turn are used to gain the support of “swing” voters.

I propose a model which integrates elements from Baron, Grossman and Helpman, and Congleton above. As in Baron and Grossman and Helpman, interest groups seek to influence the policy positions of political candidates. In particular, special-interest groups located at the extremes of a unidimensional policy space present candidates with contribution schedules which map all feasible candidate policy locations to a specific level of contribution. As in Congleton, candidates use contributions in an effort to convince voters that their policies map to favorable outcomes. The technology by which campaign spending influences voter beliefs subsumes any need to distinguish between informed and uninformed voters. The present model thus retains the basic tension between centripetal and centrifugal forces. Candidates seek to exactly balance these at the margin. The result is that even purely office-oriented candidates may diverge, and policy-oriented candidates may adopt positions more extreme than their own ideal points.

In contrast to the authors above, I assume exogenous special-interest behavior with the result that I can characterize candidate behavior with great richness. Candidate reaction functions depend in a complex but systematic way on underlying parameters describing the special-interest contribution schedules, the campaign technology, and the candidates’ own
preferences. In general, the nature of the strategic interaction between the candidates — whether a candidate responds to her opponent’s movement towards the expected median voter by doing the same or the opposite — depends on where the candidate’s best response is relative to her ideal point. In addition, for certain parameter combinations, the reaction functions are discontinuous so that a small change in opponent location may cause a candidate to make a large change in her own location.

A second benefit of the present framework is its generality. A wide range of models can be interpreted as specific instances including the ideas of informed versus uninformed voters, office-seeking versus policy-oriented candidates, and two-stage election processes. The model naturally generalizes into multiple dimensions where considerations inherently missing from single-dimension competition can be introduced.

The paper is organized as follows: Section 2 introduces the formal model. Candidates are assumed to have both office-seeking and policy-oriented motivations. The function describing the median voter’s beliefs over the mapping from policies into outcomes is implicit in his reduced-form policy ideal point. Candidates’ campaign spending affects the probability distribution from which these ideal points are drawn. Section 3 presents analytical and numerical results. Equilibrium location choices range anywhere from full convergence (i.e. location at the expected median voter) to full divergence (i.e. location at the boundary of the allowable strategy space). Comparative statics are presented for changes in special-interest group behavior, campaign technology, and candidate preferences. Section 4 includes a discussion of special cases of the model, various extensions, and empirical implications. Section 5 concludes.

2 A One-Dimensional Model of Candidate Location

2.1 The Actors

Candidates

The general framework is a two-party electoral system composed of a left party and a right party. Candidates are motivated both by the desire to hold office per se and by the desire to achieve policy realizations that most closely match their own ideals. Using an expected-utility framework with $X_L \in [-1, 0]$ and $X_R \in [0, 1]$ representing candidates’ committed policies, the office component of candidates’ preferences is simply the probability that they
are elected.

\[ U_{R, Office} = P(X_L, X_R) = \text{Probability R elected} \quad (1) \]

\[ U_{L, Office} = 1 - P(X_L, X_R) = \text{Probability L elected} \]

The policy component of candidate preferences is the expected utility loss from deviations in actual policies from the candidates’ ideal policies, \( \theta_L \in [-1, 0] \) and \( \theta_R \in [0, 1] \):

\[ U_{R, Policy} = P(\cdot) \cdot U_R(X_R) + (1 - P(\cdot)) \cdot U_R(X_L) \]
\[ U_{L, Policy} = P(\cdot) \cdot U_L(X_R) + (1 - P(\cdot)) \cdot U_L(X_L) \quad (2) \]

\[ U_k(X_k) = -\left( \theta_k - X_k \right)^2 \quad k, \hat{k} = L, R \quad (3) \]

Finally, overall candidate utilities are simply a weighted sum of the office-holding and policy components:

\[ U_R = \phi P(\cdot) + (1 - \phi) (P(\cdot) U_R(X_R) + (1 - P(\cdot)) U_R(X_L)) \]
\[ U_L = \phi (1 - P(\cdot)) + (1 - \phi) (P(\cdot) U_L(X_R) + (1 - P(\cdot)) U_L(X_L)) \quad \phi \in [0, 1] \quad (4) \]

Here \( \phi \) captures the relative weight a candidate places on office-holding relative to policy realizations. \( \phi \) equals one when a candidate cares solely about winning office and is indifferent among various policy realizations. At the other extreme, \( \phi \) equals zero when a candidate values office holding only as a means to achieving a favorable policy realization.5

Candidates face the budget constraint that their campaign spending, \( S_k \), be no greater than the contributions which they receive from special-interest groups, \( C_k \).

\[ S_k \leq C_k \quad k = L, R \quad (5) \]

5The limitation that L locates on \([-1,0]\) and R locates on \([0,1]\) is made in order to ease exposition. The outer boundaries are not particularly troublesome as they can be interpreted as the boundary of the policy space. More restrictive in terms of content is the limitation of each of the candidates to a half space bounded at zero. Removing this assumption both encumbers algebra by necessitating extra steps to avoid equilibrium policies in the range of complex numbers and also raises thorny issues regarding the specification of special-interest contribution schedules such as whether the special interests are willing to make simultaneous contributions to both of the candidates. Also avoided is potentially perverse behavior in which the L and R candidates reverse positions in the policy space. The cost of the assumption is illustrated by the case in which a purely office-oriented R candidate faces an extremist, policy-oriented L. Here it is quite likely that if L chooses an extreme location (for instance \( X_L = -1 \)), R will desire \( X_R < 0 \) even if such a location results in her receiving no campaign contributions. Analogous to the Hotelling model of location on a boardwalk, R has a motive to “box in” L. But as long as R retains a strong policy orientation, the “boxing in” motive will be tempered by R’s desire to locate near her own ideal. Nevertheless, in interpreting the reaction curves of the candidates in the results section below, we should realize that when a candidate’s best response is a locational choice of zero, in fact under a more general specification the candidate may choose to locate on the “wrong” side of zero.
Given the static nature of the present model and the lack of alternative use of campaign funds, (5) will in fact bind with equality.

**Special Interests**

There exist two special-interest groups, $PACL$ with ideal policy $\theta = -1$ and $PACR$ with ideal policy $\theta = 1$. Modeling special-interest ideal policies as extreme relative to those of candidates follows Baron (1994) and finds empirical support in Poole and Romer’s (1985) study of PAC contributions to candidates in the 1980 elections for the U.S. House. The theoretical assumption of extreme special-interest groups can be justified as deriving from the problem of free-riding which would be encountered in organizing a dense population of moderate interests. (Mancur Olson, 1965). As in Grossman and Helpman (1994, 1996), each of the special-interests makes known to the candidates a contribution schedule for each feasible candidate policy position. In contrast to Grossman and Helpman, who endogenously model the contribution schedules, I assume exogenous, continuous and twice-differentiable contribution schedules, $C_L(X_L)$ and $C_R(X_R)$:

$$
C_L = I(X_L \leq 0) \cdot \hat{\eta}_L \cdot (-X_L)^{\beta_L} \\
C_R = I(X_R \geq 0) \cdot \hat{\eta}_R \cdot (X_R)^{\beta_R}
$$

(6)

Candidates receive larger contributions as they move away from the center and toward the respective special-interest bliss points. $\beta_k$ parameterizes the marginal return — measured in contributions — to candidate divergence. Thus for $\beta_k > 1$, candidates receive increasingly larger marginal contributions as they diverge; for $\beta_k < 1$, they receive increasingly smaller marginal contributions as they diverge. $\beta_k$ equal to zero is equivalent to the electoral-motive case where special-interest contributions are not contingent on candidate location.

$\hat{\eta}_k$ parameterizes the level return to candidate divergence. Several interpretations are possible. Well-funded and powerful special interests are presumably associated with large values of $\hat{\eta}_k$. So, too, would special interests with particularly intense preferences under a more general interpretation of special-interest donations as capturing activism. $\hat{\eta}_k$ can also be seen as capturing government legislation regarding campaign contributions such as limits on contributions (thereby lowering $\hat{\eta}_k$) and government matching funds (raising $\hat{\eta}_k$). $\hat{\eta}_k$ equal to zero is equivalent to the special case where special interests do not participate in the electoral process.
The indicator functions in (6) capture the restrictions that $PACL$ not give to the R candidate and that $PACR$ not give to the L candidate.$^6$

**Voters and the Probability of Election**

The model relies on probabilistic voting with campaign spending impacting the distribution function over voter ideal points. In particular, I assume there exists a median voter with ideal point, $\theta^m$, which is distributed according to $F(\theta^m; S_L, S_R)$ where $S_L$ and $S_R$ represent campaign spending by the left and right candidates, respectively. For the sake of simplicity, I am following Alesina and Rosenthal (1995, chapter 4) in employing a reduced-form specification in which the distribution function describes the ideal point of a median rather than an arbitrary voter. However, given that for any distribution over voter preferences, probabilistic voting induces a binomial distribution over vote outcomes, and given that the binomial distribution is nicely approximated by the normal distribution, little generality is lost so long as $F(\cdot)$ is assumed to be normal.$^7$

The dependence of $F(\cdot)$ on campaign spending rests on the implicit assumption that voters do not have preferences over policy positions per se, but rather map policies into outcomes over which they do have preferences. For instance, all voters may desire a vigorously growing real economy, differing only in their beliefs on what constitutes the best policy to obtain such growth. Candidate L hires media consultants and policy analysts to make the case that a loose monetary policy will achieve vigorous growth while candidate R hires media consultants and policy analysts to make the case that in fact it is tighter monetary policy which will lead to such growth. Reasonable arguments can be made for both of these policies, and I assume that the “persuasiveness” of a candidate is proportional to her campaign spending. Voters have prior maps from policies into outcomes which lead them to form reduced-form preferences over policies; these reduced-form preferences a candidate seeks to modify by “convincing” voters that her policy position is likely to map to a more...

$^6$Implicit in (6) is the assumption that special interest contributions depend only on one or the other candidate’s positions and not on the two candidates’ positions relative to each other. Congleton (1989), in contrast, shows that wealth-maximizing special interests who take candidate positions as given will tend to make donations according to the difference in the two candidates’ positions. Certainly it is possible that influence-motivated special interests adopt a relative specification for their contribution schedules. The additively-separable specification I use here allows for much greater analytical tractability. The dependence of certain results on this assumption is discussed within the text.

$^7$Note also that the reduced-form assumption is equivalent to letting $F(\cdot)$ measure a distribution over individual voter ideal points and then assuming that policy-motivated candidates weight their policy loss functions by expected vote shares rather than probabilities of election.
positive outcome than the policy position of her opponent.8

Given that candidates are biased sources of information, the assumption that campaign spending by them influences voters’ views of the world remains problematic. Under a paradigm of rational expectations, voters should understand that any perceived differences in the quality of candidate arguments is a function solely of campaign contributions and hence reveals no information. In the Appendix, I describe a slight modification of the present setup, mostly in terms of interpretation, which makes voter response to candidate spending rational.

Ideally, I would like to explicitly model both voters’ prior maps and how these are modified by candidate campaign spending. For the present, I fall back on the less satisfying approach of arguing that voter maps are implicit in their ideal policies. Thus interpreting \( \theta^m \) as a reduced-form preference, the weakest possible assumption on the impact of campaign spending is that \( F_{SL}(\theta^m) \geq 0 \) and \( F_{SR}(\theta^m) \leq 0 \). For any given policy \( X \), campaign spending by L will not decrease the probability that the median voter favors a position to the left of \( X \). Similarly, campaign spending by R will not decrease the probability that the median voter favors a policy to the right of \( X \). In practice, I have to make much stronger assumptions on the form of the linkage between campaign spending and the distribution of voter reduced-form preferences. In particular I assume,

\[
\theta^m \sim N \left( \left( \frac{\zeta R}{\sigma^2} \right)^{\alpha} - \left( \frac{\zeta L}{\sigma^2} \right)^{\alpha}, \sigma^2 \right) \quad \zeta \geq 0, \quad \alpha \in [0, 1]
\]  

The median voter’s ideal point is drawn from a normal distribution whose mean but not variance is effected by campaign spending. Excluding campaign spending from entering the variance term is done for analytical simplicity only; under the interpretation of the normal distribution being induced from some other distribution over individual preferences, campaign spending would enter both the mean and variance terms. \( \zeta \) and \( \alpha \) parameterize the level and marginal returns to campaign spending. The level return to campaign spending captures how effectively spending shifts voter beliefs. A high level return, for instance, describes an electoral environment in which candidate spending has a large impact on voter decisions.

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8 In assuming the existence of a median voter, I am implicitly imposing single-peakedness on voters’ reduced-form utility functions. But single-peakedness seems a much less reasonable proposition when preferences are reduced form rather than primary. For instance, a voter may believe that the best policy to provide the U.S. with affordable, high-quality healthcare should rely strictly on market forces with the role of government being limited to correcting market failures. But she might also believe that a second-best policy is for the government to act as a single-payer. Policies encompassing a mishmash of market forces and government intervention the voter may perceive as mapping to less desirable outcomes than either of these extreme policies.
beliefs; a zero level return, on the other hand, captures an environment in which campaign spending is completely ineffective. The restriction of $\alpha$ to the interval $[0,1]$ captures the idea of decreasing returns to campaign spending.

A candidate’s probability of election, then, is the probability that her policy position is closer to that of the median voter than is the policy of her opponent. For candidate $R$, this probability is given by,

$$P(X_L, X_R) = \int_{x_L+X_R}^{\infty} \int_{x_L}^{x_R} \frac{1}{2\pi \sigma^2} e^{-\frac{1}{2} \left( \frac{(\theta - \mu)^2}{\sigma^2} + \frac{(\theta - \mu)^2}{\sigma^2} \right)} d\theta$$

Notice that the two parameters capturing marginal returns, $\alpha$ and $\beta_K$, and the two parameters capturing level returns, $\zeta$ and $\eta_K$, always enter jointly. As a result, I can analyze the effects of changes in each of the multiplicative products without worrying about which element, the return in terms of special-interest contributions or the return in terms of campaign technology, is actually changing. I will need to worry about whether the marginal or level return to divergence is changing for only one or for both candidates. And I will need to worry about interpretations of magnitudes of the level parameters $\zeta$ and $\eta_{L,R}$ in light of the reduced-form normalization made in (8).

## 3 Results

The simplicity of the present model belies its generality. The model can capture a wide range of real-world electoral situations and admits several leading theoretical models as special cases. But the world is a messy place with few crisp comparative statics; so too with the model. The effect of changing any given parameter value will depend on the electoral environment in which that change takes place: increasing the effectiveness of campaign spending in influencing voters’ opinions, for instance, has different effects on candidate positions when candidates are centrists versus when they are extremists. Moreover, convexity of candidate preferences implies that with complete candidate mobility, there may exist no equilibrium. Such failure of equilibrium, however, turns out to be a “rare” event.

### 3.1 Office-Oriented Candidates: Reaction Functions and Equilibrium

As a starting point, I examine reaction functions and equilibria for candidates who are purely office-oriented (i.e. $\phi = 1$). The utility of these candidates is simply their probability
of election. Given that the probability of election is also one of the major elements in the policy component of candidate preferences, understanding the forces shaping purely office-motivated candidates proves insightful in understanding the behavior of candidates more generally.

I use the standard Nash concept of equilibrium: a feasible pair of policy positions $X_L$ and $X_R$ represent an equilibrium if given $X_R$, $X_L$ maximizes L’s utility, and given $X_L$, $X_R$ maximizes R’s utility. Without loss of generality, I focus on the actions of the R candidate. For a given combination of parameter values, $(\alpha, \beta_L, \beta_R, \zeta, \eta_L, \eta_R, \theta_R)$ and a given opponent location, $X_L$, R’s first-order condition is implied by,

$$\frac{\partial U_{R,Office}}{\partial X_R} = \frac{1}{2\sqrt{2\pi}\sigma} \left( 2\zeta \eta_R \alpha \beta_R \chi_R^{\alpha \beta_R - 1} - 1 \right) \exp \left( - \frac{\left( \frac{X_L-X_R}{\sigma} - E(\theta^m) \right)^2}{2\sigma^2} \right)$$

(9)

The important point here is that since R’s opponent’s location, $X_L$ only appears in the exponential term of (9) which can never go to zero, the satisfying of R’s first-order condition does not depend on $X_L$. The implication is that the equating of the marginal benefits and costs to divergence by candidates is independent of the identity of marginal voters (i.e. the location of voters equidistant between the two candidates) whom the candidates are trying to win over. The result is robust to the choice of distribution functions over the median voter’s ideal but fragile in that it depends on separability assumptions implicit in (6) and (7).\(^9\)

Setting (9) equal to zero admits a closed form-solution for $X_R$. But this will not necessarily equal R’s location choice. Firstly, the solution to R’s first-order condition may give a location which minimizes her probability of election rather than maximizes it. R’s second-order condition (not shown) implies that this will be the case whenever $\alpha \beta_R > 1$. Secondly, even with $\alpha \beta \leq 1$, the solution to R’s first-order condition will lie above the most divergent feasible location, $X_R = 1$, whenever $\zeta \eta_R > \frac{1}{2\alpha \beta_R}$. In both of these contingencies, R will choose to locate at either $X_R = 0$ or $X_R = 1$, whichever gives her a higher probability of election. Finally, it can be shown that $P(X_R = 1) < P(X_R = 0)$ as $\zeta \eta_R < \frac{1}{2}$. R’s locational choice can thus be summarized by,

\(^9\)The mathematical intuition is that both costs and benefits are proportional to $f \left( \frac{X_L + X_R}{2} \right)$, the density function measured at the “marginal” voter. L’s location would enter R’s choice if R were to consider the effect the change in her location would have on L’s location. But this possibility is ruled out by the assumption of Nash behavior.
\[ X_{R,\text{Office}} = \begin{cases} 
0 & \alpha\beta_R > 1, \zeta\eta_R < \frac{1}{2} \\
1 & \zeta\eta_R > \max\left(\frac{1}{2\alpha\beta_R}, \frac{1}{2}\right) \\
(2\zeta\eta_R\alpha\beta_R)^{-\frac{1}{1-\alpha\beta_R}} & \text{otherwise}
\end{cases} \] 

(10)

Note that in no case does R’s best response depend on \(X_L\), \(\beta_L\) or \(\eta_L\). Hence,

**Result 1:** With special-interest contribution schedules and campaign technology additively separable in candidate locations and spending, purely office-oriented candidates’ best response functions are independent of opponent location and any factors that do not directly affect their own return to divergence.

Figure 1 summarizes (10) on \(\alpha\beta_R \in [0, 2] \times \zeta\eta_R \in [0, 2]\). The two “Maximal Feasible Divergence” regions, I and II, differ in that as R diverges from \(X_R = 0\) to \(X_R = 1\), R’s probability of election is first decreasing and then increasing in region I but monotonically increasing in region II. In regions III and IV, R chooses to locate at \(X_R \in [0, 1]\) as given by (10). In region III, increases in the marginal return to divergence, \(\alpha\beta_R\), cause R to diverge away from zero; in region IV, increases in the marginal return cause her to converge towards zero. In both regions, increases in the level return to divergence, \(\zeta\eta_R\), cause R to diverge away from zero. The derivation and exact mathematical expressions are deferred to the appendix.\(^{10}\)

**Result 2:** For purely office-oriented candidates locating intermediate between their feasible extremes:  

A. Candidates will diverge in response to an increase in the marginal return to divergence whenever the magnitude of the level return to divergence is large relative to the marginal return to divergence; otherwise they will converge.  

B. Candidates always diverge in response to an increase in the level return to divergence.

The location which maximizes R’s probability of election, (10), along with Result 2 are particularly important when trying to parameterize the model to accurately capture what is going on in the real world. As is sketched out in Figure 1, for a large range of parameter

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\(^{10}\)R’s locational choice is continuous with respect to both sets of parameter values on the borders between all regions except for that which divides I from V. Here, \((\alpha\beta_R \geq 1; \zeta\eta_R = \frac{1}{2})\), R’s probability of election is equal at both \(X_R = 0\) and \(X_R = 1\) and hence R is indifferent between the two; but for slight perturbations in the level return to divergence, \(\zeta\eta_R\), R’s probability will be higher at one or the other extreme. Where all five regions converge, \((\alpha\beta_R = 1; \zeta\eta_R = \frac{1}{2})\), it can be shown that R’s probability of election is everywhere equal on \(X_R \in [0, 1]\) so that R’s locational choice is completely indeterminate.
values R’s probability of election is maximized at her most extreme feasible policy location. Yet it seems that rarely in the real world are extreme candidates electorally viable. One explanation consistent with the model is that the real-world level return to divergence is very low; this would be the case, for instance, if the financial resources which special interests could muster were small relative to that which would be necessary to persuasively sway large portions of the electorate. Under this interpretation, endogenizing special-interest behavior would presumably result in the special interests’ choosing contribution schedules with relatively low marginal returns to divergence: the lower the level return to divergence, the lower the marginal return which maximizes candidate divergence. A second explanation is that even when special interests have the ability to offer very high level returns to divergence, policy preferences keep candidates from moving very far out increasing-marginal-return contribution schedules. A third explanation, is that we do see extreme candidates in the real world, only on issues that few people care about and therefore, for which, the level return to divergence may be quite high. The various explanations are worth keeping in mind in interpreting the results for policy-oriented candidates below.

Returning to the initial question of equilibrium, with competition between two purely office-seeking candidates, Result 1 immediately implies that there will always exist an equilibrium. Moreover, except for parameter values that lie on the border between Regions I and V of Figure 1, the equilibrium will be unique.

### 3.2 Purely Policy-Motivated Candidates: Reaction Functions

For this and the next four subsections, I turn to the case of purely policy-motivated candidates (i.e. \( \phi = 0 \)). Unlike the case of purely office-seeking candidates above, policy motivated candidates’ best response functions will in general depend on their opponent’s location. For a given combination of parameter values, \((\alpha, \beta_L, \beta_R, \zeta, \eta_L, \eta_R, \theta_R)\) and a given opponent location, \(X_L\), R’s first-order condition is implied by,

\[
\frac{\partial U_R}{\partial X_R} = \frac{1}{2\sqrt{2\pi\sigma}} \left( 2\zeta \eta_R \alpha \beta_R X_R^{\alpha \beta_R - 1} - 1 \right) \exp \left( -\frac{(X_L + X_R - E(\theta^m))^2}{2\sigma^2} \right) \]

\[
\cdot \left( (\theta_R - X_L)^2 - (\theta_R - X_R)^2 \right) + \left( \int_{\theta^m}^{\infty} f(\theta^m; \cdot) \, d\theta^m \right) \cdot 2(\theta_R - X_R)
\]

The first set of terms captures the change in R’s probability of election multiplied by the utility differential to R of winning the election and implementing her policies rather
than those of her opponent. The second set of terms captures the change in R’s utility multiplied by the fixed probability of election. In general, (11) does not admit a closed-form solution. Nor does its solution necessarily represent a solution to R’s choice problem since a second-order condition, $\frac{\partial^2 U_R}{\partial (X_R)^2} \leq 0$, must also be satisfied. As with purely office-oriented candidates, R’s best response may lie at the extremes of the feasible strategy space.

When, then, will candidates locate moderate relative to their ideals and when, extreme relative to them? From the previous section on purely office-seeking candidates, we know that so long as the marginal return to divergence is decreasing ($\alpha \beta_R < 1$), there exists a unique location, $X^*$, that maximizes a candidate’s probability of election; moreover quasiconcavity implies that candidates can only increase their probability of election relative to what it would be at their ideal, $\theta$, by moving away from $\theta$ in the direction of $X^*$. As a result,

$$\alpha \beta_R < 1 \Rightarrow X_R > \theta_R \Rightarrow (2\xi R \alpha \beta_R)^{\frac{1}{1-\alpha \beta_R}} > \theta_R$$

result 3: When the marginal return to divergence is decreasing, candidates will locate in the same direction relative to their ideal point as the policy which maximizes their probability of election.

When the marginal return to divergence is increasing, no single criteria characterizes when candidates converge or diverge relative to their ideal.

Further analytical results regarding both the slopes of candidate reaction functions and the directions in which they shift in response to changes in parameter values are presented in subsections below. These analytical results are limited, however, in that they hold only under symmetric parameter values and only in the neighborhood of equilibria. More generally, we would like to know what candidate reaction functions look like under non-symmetric parameter values (and hence away from symmetric equilibria). Numerical solutions to the maximization problem implied by (2), (3), and (8) readily sketch out such reaction curves. Figure 2 provides four examples. Here, as well as in Figures 3 and 4 below, I assume candidates with ideal points at $\theta_{L,R} = \pm 0.15$ and a standard deviation of the median voter’s ideal of $\sigma = 0.2$. This combination implies that in the absence of campaign spending, with 55% probability the median voter’s ideal will fall between that of the two candidates and with 45% probability, it will fall more extreme than either of the two candidates. Note also that for the moment, I have assumed symmetric parameter values.
Panel A represents an environment where both the marginal ($\alpha \beta_{L,R}$) and level ($\zeta \eta_{L,R}$) returns to candidate divergence are relatively low. Candidate best response functions lie everywhere intermediate between candidate ideal points and zero. Thus regardless of L’s actions, R will converge from her ideal point towards zero and vice versa. The choice by a candidate to enhance her election probability by converging from her ideal towards zero I will term a “moderation-based” strategy. The panel B electoral environment differs from that in Panel A in that both the marginal and level return to divergence are higher. In this case, candidate best response functions lie everywhere more divergent than candidate ideal points: regardless of L’s actions, R will diverge from her ideal point away from zero and vice versa. The choice by a candidate to enhance her election probability by diverging from her ideal point away from zero I will term a “money-based” strategy.

Panels C and D introduce increasingly higher values of the marginal and level returns to divergence. In Panel C, candidates once again adopt money-based strategies. The resulting reaction functions are sufficiently steep that three equilibria emerge: a relatively moderate equilibrium (though divergent relative to candidate ideals), an intermediate equilibrium, and an extreme equilibrium in which both candidates adopt their most divergent feasible policy locations.

In Panel D, as their opponent initially moves away from zero, the candidates pursue moderation-based strategies and are eventually constrained to their most convergent feasible location. The moderation strategies result in a first equilibrium convergent relative to candidate ideals. In terms of Figure 1, note that the presence of increasing marginal returns to divergence place the candidates in either region I or V; either way, probabilities of election are quasiconvex with respect to candidate locations so that points interior to [0, 1] may actually minimize probabilities of election. In Panel D, candidates eventually respond to opponent divergence by abandoning their moderation strategies in favor of money-based strategies. For $X_L$ equal to -.4125, R chooses to locate at $X_R = 0$. But for $X_L$ equal to -.4250, R “jumps” all the way to $X_R$ equal to 0.4710. This sort of discontinuity in candidate reaction functions — the switching between moderation and money-based electoral strategies — is the source of the possibility that the model will fail to admit an equilibrium.

Footnote 11: Without the restriction that candidates locate in their own half of the strategy space, we might expect R to espouse left-of-center policies and R to espouse right-of-center ones.
It is also a source of multiple equilibria: in Panel D there exists a second equilibrium with both candidates’ adopting their most divergent feasible policy locations.

A necessary condition for reaction functions to exhibit discontinuities as in Panel D is that the marginal return to divergence be increasing ($\alpha_\beta_k > 1$). Otherwise Result 3 implies that candidates would move in only one or the other direction relative to their ideal policy. Beyond this, however, I am unable to enumerate definitive conditions which lead to discontinuous reaction functions. Based on the numerical simulations, discontinuous reaction functions are more common the higher is the marginal return to divergence. This makes sense as the convexity of a candidate’s probability of election is increasing in $\alpha \beta$. An additional common feature is that candidate ideals are located closer to the extreme which offers the lower probability of election – for instance candidates with ideal policies relatively close to zero in electoral environments characterized by a high level return to divergence, i.e. Region I of Figure 1. As long as their opponent’s policy is moderate “enough”, candidates are willing to accept a lower probability of election in return for a higher ex post utility should they win office. But as their opponent policies become more extreme and hence onerous, candidates become willing to sacrifice the ex post utility associated with their own election in order to minimize the probability that their opponent’s policy is realized. Result 4 summarizes:

Result 4: For purely policy-oriented candidates with an increasing marginal return to divergence, reaction functions may exhibit discontinuities.

3.3 Purely Policy-Motivated Candidates: Equilibrium

The discontinuity of candidate reaction functions in the last panel of Figure 2 highlights that for policy-motivated candidates (and by extension for candidates with mixed motivations), there may in fact not exist a Nash equilibrium policy combination. A systematic search of the parameter space $\alpha_\beta_L \times \alpha_\beta_R \times \zeta \eta_L \times \zeta \eta_R \in [0,2]^4$, however, shows that failure of equilibrium is a relatively rare event. Table 1 summarizes the result of such a search. Over approximately 80% of the parameter space, R’s reaction function is continuous.\(^{13}\)

\(^{12}\)While this latter explanation surely captures much of the dynamics underlying discontinuous reaction functions, it fails to capture second-order effects and so is incomplete. Over a very small portion of the parameter space, R’s reaction function actually has two discontinuities. For moderate L positions, R adopts positions slightly divergent from her ideal. As L diverges, R eventually jumps to $X_R = 0$. As L continues to diverge, R eventually jumps back to extreme positions herself.

\(^{13}\)In addition to rounding, “approximately” is meant to connote that the numerical routines used to derive Table 1 while highly accurate are nevertheless imperfect. In particular, I would assert that humans seem to
Over approximately 63% of the parameter space, both candidates’ reaction functions are continuous so that at a minimum there exists one equilibrium. Failure of equilibrium, in contrast, occurs over only about 1.33% of the parameter space. Indeed, a unique equilibrium is by far the norm, occurring over approximately 94% of the space. Two and three equilibria occur about 2.6% and 1.7% of the time respectively. Finally, very rarely (about 0.1% of the time) there exist four equilibria. While Table 1 is based on candidates with symmetric ideal points, qualitatively similar results are obtained when ideal points are not symmetric.

Table 1 about here

Result 5: For purely policy-motivated candidates: A There may exist no equilibrium. B There may exist multiple equilibria. C Over the majority of the parameter space, there exists a unique equilibrium.

How, then, to interpret the non-existence of an equilibrium? The presence of multiple equilibria? Shepsle (1972) suggests that the “‘search for equilibria’ may not be the appropriate theoretical posture — that instability and the absence of equilibrium (in the dominance sense) may be the rule rather than the exception, and that the interesting behavior of the model is the strategic adaptation of actors to these nonequilibrating contingencies.” More modestly, I would argue that spatial modeling assumes a degree of candidate mobility in the issue space that is seldom realized in practice. Few are the blank-slate candidates who can credibly adopt the best response position to any current electoral environment. More commonly, and especially for incumbents, electoral environments will shape the direction and magnitude of change that candidates make from some status quo position. In the long run (i.e. over several elections), the question of instability remains. But within the context of the current model, failure of equilibrium is sufficiently an exception rather than the rule that it seems reasonable to hypothesize that unmodeled forces — for instance endogenous special-interest behavior, endogenous candidate preferences, or threats of third-party entry enjoy a sizable comparative advantage over computers in judging whether two, possibly discontinuous, lines intersect.

14 Failure of equilibrium, however, would probably occur much more commonly under a relative specification of campaign contribution schedules. See Congleton (1989).
3.4 Policy-Motivated Candidates: Strategic Interaction

Focusing on candidate reaction functions rather than equilibria, a first basic question regards the nature of the strategic interaction between the two candidates: as candidate L adopts positions increasingly away from zero, how does candidate R respond? Does she diverge also, moving towards +1 as L moves towards -1? Or does R converge, moving towards 0 as L moves away from it? In terms of the reaction functions themselves, the question is equivalent to asking whether their slopes are negative or positive. The answer, it turns out, depends on whether a candidate has converged or diverged relative to her own ideal policy. As a starting point, consider the strategic interaction in the neighborhood of a symmetric equilibrium (i.e. in the neighborhood of an equilibrium given $\beta_L = \beta_R, \eta_L = \eta_R, \theta_L = \theta_R$). Under symmetry, $X_L = -X_R$, and R’s first-order condition reduces to,

$$\frac{\partial U_R}{\partial X_R}\bigg|_{\text{symmetry}} = \left(\frac{2\theta_R X_R}{\sqrt{2\pi}}\right) \left(2\zeta \eta_R \alpha \beta_R^{\alpha \beta_R^{-1}} - 1\right) + \theta_R - X_R$$

(13)

Totally differentiating R’s general first-order condition (11) with respect to $X_L$ and $X_R$ and using (13) to substitute yields the change in $X_R$ in response to a change in $X_L$ in the neighborhood of a symmetric equilibrium:

$$\frac{\partial X_R}{\partial X_L}\bigg|_{\text{symmetry}} = \frac{\theta_R - X_R}{\theta_R - \left(\frac{2\theta_R X_R}{\sqrt{2\pi}}\right)} > 0 \text{ as } X_R < \theta_R$$

(14)

At an internal equilibrium, the bracketed term in the denominator of (14) is always negative and hence the overall sign is the same as the sign of the numerator. The numerator is positive whenever $X_R$ is intermediate between $\theta_R$ and zero and is negative whenever $X_R$ is greater than $\theta_R$. Thus when R’s best response is to locate at a position more moderate than her ideal, she responds to L’s diverging away from zero (towards -1) by converging towards zero. In contrast, when R’s best response is to locate at a position more extreme than her ideal, she responds to L’s diverging away from zero by also diverging away from zero (towards +1). Hence, under moderation-based strategies, R responds to increased spending by L and the increased divergence that funded this by adopting ever more moderate policy.

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15On the other hand, long-run electoral cycles may be consistent with real world behavior. See, for instance, Schlesinger (1986), and Alesina and Rosenthal (1995).
positions. Under the money-based strategies, she responds to increased spending by L by increasing her own spending and divergence.

The analytical result implied by (14) is readily apparent in the various panels of Figure 2. At the two moderate-based strategy equilibria, (Panels A and D), candidate reaction functions are positively sloped. At the three money-based strategy internal equilibria, (Panels B and C), the reaction functions are negatively sloped. In addition, Panels C and D contain money-based strategy boundary equilibria. Here, the vertical and horizontal reaction curves of L and R, respectively, imply that in the neighborhood of such equilibria, neither candidate responds to small changes in position by the other.

Away from the symmetric equilibria, reaction functions for the most part continue to be positively sloped where they lie moderate relative to a candidate’s ideal and negatively sloped where they lie extreme relative to a candidate’s ideal. But there are exceptions. In Figure 2 Panel A, for extreme opponent positions, candidate reaction functions are negatively sloped despite lying moderate relative to candidate ideals. Figure 4, Panels C and D provide corresponding examples of reversals in sign of the slope of R’s reaction curve when her best response is to locate extreme relative to her ideal. These sign reversals are nevertheless relatively uncommon and always occur in response to extreme opponent locations.

**Result 6**: A For a purely policy-motivated candidate in the neighborhood of an internal symmetric equilibrium, if her best response is to locate extreme relative to her ideal point, she will respond to her opponent’s diverging by doing the same; if her best response is to locate moderate relative to her ideal point, she will respond to her opponent’s diverging by doing the opposite. B This pattern tends to hold for reaction functions more generally.

### 3.5 Comparative Statics

As suggested by Shepsle’s conjecture that the search for equilibria may be the wrong theoretical posture, the comparative statics which follow all relate to shifts in reaction functions rather than shifts in equilibria. The justification is more pragmatic than ideological: the nature of shifts in equilibria will always depend on both the direction of shifts in the associated reaction functions and the qualitative nature of these reaction functions (i.e. their slope, continuity, and location relative to ideal points, etc.). Given the four possible combinations of shifts in the L and R reaction functions multiplied by the many possible combinations of the qualitative properties of the L and R reaction functions, a discussion of the compara-
tive statics of equilibria is impractical. The impact of changes in parameter values on any specific equilibrium, on the other hand, will necessarily follow from the shifts in the specific associated reaction functions.

A helpful distinction to make with regard to the results that follow are those that relate to candidate reaction functions along their entire length versus those that relate to the reaction functions only in the neighborhood of a symmetric equilibrium. In general I will focus on the shifts in candidate reaction functions along their entire length. Given that such “shifts” will at times be more properly categorized as “rotations” or “bifurcations”, they lack the crispness usually associated with comparative statics. Therefore, where the assumption of symmetry allows me to analytically derive sharp comparative statics, I will also discuss shifts in the reaction functions in the neighborhood of symmetric equilibria.

**Changes in the Level of Uncertainty**

A first, brief comparative static regards the effect of changes in the level of uncertainty, $\sigma$. From R’s first-order condition under symmetry, (13), it immediately follows that in the neighborhood of a symmetric equilibrium, increases in uncertainty cause R’s reaction function to shift towards her ideal. More generally, an expansion of R’s generic first-order condition, (11), shows that as uncertainty becomes infinite, R’s first-order condition is satisfied only by her locating at her ideal policy. The intuition for this latter case is simply that with infinite uncertainty, the probability of election is everywhere the same so that R might as well adopt the policy yielding the highest utility if she is elected.

**Result 7**: A For purely policy-oriented candidates in the neighborhood of a symmetric equilibrium, increases in uncertainty cause reaction functions to shift towards candidate ideals. B As uncertainty becomes infinite, purely policy-oriented candidates adopt their most-preferred policies.

**Changes in Marginal Buying Power**

As already discussed, each of the candidates faces an effective marginal return to divergence which is the multiplicative product of the marginal campaign contributions received, $\beta_L$ and $\beta_R$, with the marginal return to campaign spending, $\alpha$. Figure 3 shows the comparative statics of variations in the combined marginal returns to divergence. An important caveat here is that to the extent that the L and R interest groups represent unitary Stackelberg actors, the products $\alpha\beta_L$ and $\alpha\beta_R$ should be considered special-interest choice variables.
The comparative static analysis herein would then be capturing the second stage of a two-stage game.

**Figure 3 about here**

Panel A shows the effect of increases in both $\alpha \beta_L$ and $\alpha \beta_R$ within an electoral environment characterized by a low level return to divergence, $\zeta \eta_{L,R} = 0.3$. For all four marginal return parameters shown, candidates adopt policy locations between their ideal and zero regardless of opponent location. Result 3 along with Figure 1 imply that this will necessarily be the case for $\alpha \beta \leq 1$. As $\alpha \beta_L$ and $\alpha \beta_R$ increase from 0.1 to 0.4 (curves “a” and “b”), we are moving horizontally through region III of Figure 1. Here the policy location that maximizes R’s probability of election is increasing with $\alpha \beta$ which explains the shift up from “a” to “b”. As the marginal returns continues to rise to $\alpha \beta_{L,R} = 0.95$, we are now moving through region IV of Figure 1 and the location maximizing R’s probability of election is decreasing with $\alpha \beta$; hence the shift down from “b” to “c”. Finally, for $\alpha \beta_{L,R} > 1$, we have entered region V of Figure 1; in such an electoral environment, a policy-oriented candidate with a relatively moderate ideal policy quickly responds to increased opponent divergence by adopting the probability-maximizing location of maximal feasible convergence.

In Panel B, the higher level return to divergence, $\zeta \eta_{L,R} = 0.6$, implies that increases in $\alpha \beta$ move us horizontally through regions III, II, and I of Figure 1. As $\alpha \beta_L$ and $\alpha \beta_R$ increase from 0.1 to 0.4 to 1.0, (curves “a” ,“b”, and “c”), the location maximizing the candidates’ probabilities of election shifts from $\pm 0.09$ to $\pm 0.29$ to $\pm 1$ in turn causing the continual shift upward from the moderation-based strategy of curve “a” to the money-based strategies of curves “b” and “c”. For $\alpha \beta_{L,R} > 1$, candidates’ probabilities of election are quasiconvex with respect to their policy locations and so analytical predictions are difficult to make. Curve “d” illustrates what happens for $\alpha \beta_{L,R} = 1.6$. Under this parameterization, $X_{L,R} = \pm 0.34$ minimize candidates’ probabilities of election while $X_{L,R} = \pm 1$ maximize them. Rather than maximizing their probabilities of election, however, the purely policy-motivated candidates pursue moderation-based strategies, which yield them greater ex-post utilities should they be elected.

Panels C and D highlight that each candidate’s reaction function takes into account the returns to divergence not just for herself, but also for her opponent. In these panels, the marginal return to divergence increases only for candidate R. The effects on R’s reaction
curves of shifts in $\alpha \beta_R$ look identical to the effects of shifts in both $\alpha \beta_L$ and $\alpha \beta_R$ in Panels A and B above. The big difference is the response of L’s reaction function. Though $\alpha \beta_L$ remains constant, L’s reaction curves shift in response to changes in $\alpha \beta_R$ since these changes imply that each R position becomes associated with a different amount of R campaign spending. As captured in the two panels, increases in $\alpha \beta_R$ cause L’s reaction function to shift toward her ideal point, $\theta_R$. When L is playing a moderation strategy (Panel C), this shift represents a move away from zero whereas when L is playing a money-based strategy (Panel D), it represents a move towards zero.\textsuperscript{16} Result 8 summarizes:

**Result 8:** For purely policy-oriented candidates: A With nonincreasing marginal returns to divergence, symmetric changes in both candidates’ marginal return to divergence cause reaction functions to shift in the same direction as the associated shift in probability-maximizing policy locations. B Increases in an opponent’s marginal return to divergence cause candidate reaction functions to shift toward candidate ideals.

**Changes in Level Buying Power**

The three parameters that calibrate the level return to campaign spending, $\zeta$, $\eta_L$, and $\eta_R$ capture a wide range of phenomena including voters’ susceptibility to influence by campaign spending, the relative financial strengths of the two special interests, and any campaign finance laws which limit or subsidize campaign activities. Intuitively, changes in the level return to campaign spending should have two effects. The higher the level return, the greater the reward to diverging and so we should expect reaction curves to shift away from zero. For purely office-oriented candidates, this is the only effect and reaction functions do everywhere diverge with increases in the level return. But for policy-oriented candidates, an increase in the level return can be used to maintain a constant probability of election while allowing the candidate to move closer to her ideal policy. Depending on the candidate’s location relative to her ideal, this latter effect may reinforce or counteract the first effect toward greater divergence.

Analytically, comparative statics deliver unambiguous results only for shifts in reaction functions in the neighborhood of a symmetric equilibrium. Starting from a symmetric equilibrium, consider first increases in the level return to divergence for both candidates. In

\textsuperscript{16}The explanation here is purely mathematical, namely that for constant $\zeta \eta_R$, increases in $\alpha \beta_R$ are associated with a downward shift in the R’s gross return to divergence, $\zeta \eta_R X_R^{\alpha \beta_R}$, for $X_R \in [0, 1]$. The net result is similar to a decrease in R’s level return to divergence, the comparative static for which I discuss in the subsection immediately below.
such a case, the intuition that higher level buying power is associated with more divergence holds: increases in both $\zeta_{L}$ and $\zeta_{R}$ (which I will connote analytically by changes in only the non-subscripted “$\zeta$”) are associated with local shifts away from zero of the L and R reaction functions:

$$\frac{\partial X_{R}}{\partial \zeta} \left|_{\text{symmetry}} \right. = \frac{2\sqrt{\pi} \eta_{R}^{\alpha} \beta_{R}^{\theta} R^{\alpha} R^{\beta}}{-\left( \frac{\partial^{2} U_{R}}{\partial X_{R}^{2}} \right)} > 0 \quad (15)$$

Figure 4, Panels A and B show two examples of increasing the level returns for both candidates. As the level returns get progressively higher (moving from curves “a” to curves “f” in both panels), $X_{R}(X_{L})$ shifts up and $X_{L}(X_{R})$ shifts left in the neighborhood of their intersection with each other. Usually this also leads the resulting symmetric equilibrium to move away from zero. But it need not do so; the middle equilibrium generated by curves “e” in Panel B illustrates a case where a small increase in the level returns for both candidates causes an equilibrium to shift towards zero. Nor do the shifts away from zero hold along the entire lengths of the reaction curves. In Panel A, for very moderate $X_{L}$, R’s reaction curve “f” lies below curve “e” despite being associated with a higher level return to divergence. Similarly, Panel B curves “a” through “d” show that where candidates play moderation strategies, increases in level buying power can cause reaction curves to shift towards zero along portions of their length.

Result 9: **A** For purely policy-oriented candidates in the neighborhood of a symmetric equilibrium, increases in both candidates’ level return to divergence cause reaction functions to shift away from zero. **B** More generally, candidate reaction functions, and equilibrium policy locations respond non-monotonically to increases in both candidates’ level buying power.

I next examine the comparative statics of shifts in the level return to divergence faced by only one of the two candidates. Once again, analytical comparative statics can be definitively signed only in the neighborhood of a symmetric equilibrium. Deferring the mathematical expressions to the Appendix, the result is that so long as candidates are playing moderation-based strategies and hence are located intermediate between their ideal point and zero, the two possibly counteracting forces already discussed act in the same direction. Increases in R but not L’s level return lead R to diverge from zero both to
increase her probability of election and to move closer to her ideal policy. Increases in $L$ but not $R$’s level return cause $R$ to move closer to zero and hence away from her ideal point. When, however, the candidates are playing money-based strategies, their responses to changes in one or the other’s level return can be in either direction.\textsuperscript{18}

Figure 4 about here

Figure 4, Panels C and D show the effect of increases in $R$’s level return while holding $L$’s level return constant. Shifts in reaction functions along their entire length, even at non-symmetric parameter combinations, look qualitatively the same as the analytically-implied shifts in the neighborhood of a symmetric equilibrium. In Panel C, for the relatively low level returns associated with curves “a”, “b”, and “c”, ($\zeta \eta_R = 0.2, 0.3, \text{ and } 0.4$), $R$ plays a moderation-based strategy and increases in $\zeta \eta_R$ are associated with divergent shifts in her reaction function. As the level return continues to increase, $R$ shifts to a money-based strategy. At first, increases in the level return continue to be associated with divergent shifts in $R$’s reaction function, (curves “d” to “e” as $\zeta \eta_R$ goes from 0.6 to 1.0). Eventually, however, increases in $R$’s level return cause her to move toward her ideal point (curves “e” to “f” as $\zeta \eta_R$ goes from 1.0 to 2.0). For the entire range of increases in $R$’s level return, $L$ plays a moderation-based strategy and hence her reaction function continually shifts towards zero. In Panel D, increases in $R$’s level return in all cases cause $R$’s reaction function to shift away from zero and $L$’s reaction function to shift towards zero. Although not shown, continued increases in $R$’s level return do eventually cause her reaction function to shift towards zero. This reversal takes place, however, only for $\zeta \eta_R$ equal to about 3.0 and only along the portion of her reaction function corresponding to more extreme $L$ positions.

\textbf{Result 10: A} For a purely policy-oriented candidate in the neighborhood of a symmetric equilibrium, playing a moderation-based strategy is a sufficient condition for the candidate’s reaction function to shift away from zero in response to an increase in her level return to divergence and to shift towards zero in response to an increase in her opponent’s level return.\textbf{ B} This pattern tends to hold more generally. \textbf{C} For a purely policy-oriented candidate playing a money-based strategy, continued increases in her level return to divergence relative to that of her opponent tend to eventually cause the candidate’s reaction function to shift

\textsuperscript{18}(A3) shows that when a $R$’s best response is sufficiently large relative to her ideal, increases in $R$’s level buying power will cause $R$ to converge toward her ideal and zero and increases in $L$’s level buying power will cause $R$ to diverge from her ideal and zero.
Allowing for unequal level returns to candidate divergence leads to some interesting interpretations. The ratio $\zeta\eta_R/\zeta\eta_L$ can be seen as a correlate of the relative financial strength of the associated special interests: the higher the ratio, the greater the amount of money available on the right end of the political spectrum relative to that available on the left. A cautionary note here is that the transformation from the primitive parameters of the level return to divergence, $\hat{\eta}_L$ and $\hat{\eta}_R$ in (6) to their reduced forms, $\eta_L$ and $\eta_R$ in (8) implies that the actual cardinal value, $\zeta\eta_L/\zeta\eta_R$, does not necessarily capture the cardinal ratio of the level returns faced by the candidates.\footnote{Consider the case with decreasing marginal returns to campaign spending, ($\alpha < 1$), and with R’s enjoying a level advantage compared with L, ($\eta_R > \eta_L$). The point is that R’s advantage is partly offset by the diminishing returns to campaign spending so that $\frac{\eta_R}{\zeta\eta_L} < \frac{\eta_R}{\zeta\eta_L}$. To back out the primitive ratio of level returns, we need to distinguish between the components of the multiplicative products $\alpha\beta_L$ and $\alpha\beta_R$. Even then, interpretations are made all the more difficult by the suspicion that in a more general equilibrium setting, each of the reduced form products ($\zeta\eta_L, R, \alpha\beta_L, R$) is effectively a special-interest choice variable. For this reason, I do not try to map reduced-form ratios back to their primitive antecedents.}

In Figure 4 Panel C, curves “a” represent the case of equal level returns for the two candidates. The reaction functions intersect at $X_{L,R} = \pm 0.1151$ at which both of the candidates have an equal probability of election. As $\zeta\eta_R/\zeta\eta_L$ increases to 1.5 (curves “b”), the intersection shifts to $\{X_L = -0.1088, X_R = 0.323\}$ at which R’s probability of election is $P = 0.528$. As described above, the increase in R’s level return has caused her reaction function to shift away from zero and towards her ideal. Her actual campaign spending thus rises by more than the 50% increase in her level return. L’s reaction function, on the other hand, has shifted towards zero: though her level return remains unchanged, she will now raise and spend less campaign funds. At the equilibrium, R outspends L by a ratio of about 1.75 to 1. Despite this, R’s expected lead in terms of her probability of winning is only 5.6 percentage points. For $\zeta\eta_R/\zeta\eta_L = 2$ (curves “c”), the associated equilibrium is $\{X_L = -0.1022, X_R = 0.466, P = 0.563\}$. Here R outspends L by nearly 2.7 to 1. Even so, her expected lead is only 12.6 percentage points. In both cases, L moves to offset R’s spending advantage by taking electoral stands that in the absence of campaign spending will be more appealing to the expected median voter. L’s offsetting of R’s level-return electoral advantage is even more pronounced in the increasing marginal return environment of Panel D. Curves “c” here represent the case where $\zeta\eta_R/\zeta\eta_L = 17$. At the equilibrium, $\{X_L = -0.069, X_R = 0.151, P = 0.495\}$, the advantage R receives from facing an increased level return to divergence has been completely dissipated in shifting the policy debate to
the right. Despite outspending her rival by 81:1, R is nevertheless expected to win with less than even odds!

This last result – that in equilibrium, a candidate with a massive relative advantage in raising funds relative to an otherwise equal opponent is elected with less than even probability – contradicts a common-sense view of real-world electoral competition and suggests placing additional constraints on parameter values. Such calibration remains an empirical issue. A larger point, however, is that while special-interest contributions — interpreted as money, activism, or some other input — are an important determinant of electoral outcomes in the present model, they are only one of several forces at work.

Changes in Candidate Preferences

The two key components of candidate preferences are for policy-oriented candidates, their ideal policies; and for candidates more generally, the relative utility derived from policy outcomes versus from office holding per se. I examine changes to each of these in turn.

Figure 5, Panels A and B show the effect of increases in the candidate ideal points. Panel A captures a centripetal electoral environment in which the candidates’ probabilities of election are maximized at the their most convergent feasible location, zero. The result is that all candidates play moderation-based strategies, choosing always to locate between their ideal point and zero. Panel B captures an intermediate environment in which candidates’ probability of election is maximized at $\pm 0.16$. Here, candidates with ideal points more moderate than 0.16 (in particular, the candidates with ideals $\pm 0.05$ represented by curves “a”) play money-based strategies while those with more extreme views (in particular, the candidates with ideals $\pm 0.25, \pm 0.35$ represented by curves “c” and “d”) play moderation-based strategies. In both panels, increases in candidate ideal points are associated with length-wise divergent shifts in the candidates’ reaction functions.

Figure 5 about here

Figure 5, Panels C and D show the effect of varying the weight candidates place on office holding relative to achieving policy outcomes. Unsurprisingly, increasing the weight put on office-holding causes reaction functions to shift towards policy locations that maximize candidates’ probabilities of election. Panel C illustrates a set of parameter values inducing a centripetal electoral environment — one in which candidates’ probabilities of election are
maximized by locating at zero. Panel D, on the other hand, illustrates a set of parameter values inducing a centrifugal electoral environment — one in which candidates’ probabilities of election are maximized by locating at ±1. Notice that the only parameter which varies between the two panels is the level return to divergence, and this, only by a slight amount ($\eta_{L,R} = 0.49$ versus $\eta_{L,R} = 0.51$). Referring to Figure 1, this small difference is sufficient to locate Panel C’s electoral environment on the border between regions IV and V but Panel D’s electoral environment on the border between regions I and II. Also note that even when candidates place a relatively large weight on office-holding relative to policy outcomes (curves “c” in both panels, curve “d” in Panel D), the resulting reaction functions look more similar to those of purely policy-oriented candidates than to those of purely office-seeking ones.

**Result 11:** A Shifts in candidates’ ideal policies tend to cause their reaction functions to shift in the same direction. B As candidates derive increasing utility from office holding relative to policy outcomes, their reaction functions shift towards the location that maximizes their probability of election.

### 4 Special Cases, Extensions, and Empirical Implications

#### Special Cases

As I have already suggested, a number of existing spatial-voting models can be interpreted as special cases of the present model. In some cases, all that is necessary is a reinterpretation per se. To capture two-stage election processes, for instance, reinterpret the candidates as parties that must choose from the fixed positions of a number of first-stage candidates. In other cases, the specialization is trivial: a zero level return to divergence – either due to the financial impoverishment of special-interest groups or the complete ineffectiveness of campaign spending – captures a world in which special-interest politics is irrelevant. Similarly, a zero marginal return to divergence captures a world in which only the median-voter-induced centripetal force is present.

More substantively, by switching from a distribution over the expected median voter, $f(\theta^m; \cdot)$ to a distribution over all voters, $f(\theta^i; \cdot)$, a number of theoretical frameworks can be captured. The separation of the electorate into informed and uninformed voters, for instance, can be modeled by assuming that voters are drawn from two different populations: each described by its own distribution function over voter ideals and each with its own campaign technology. The “informed” voter population would be described by a very
low level return to campaign spending; informed voters’ views are difficult to sway. The “uninformed” voter population, on the other hand, would be described by a relatively high level return to campaign spending; for them, campaign spending matters a lot in forming views. In such a setup, the degree of “informedness” itself can be allowed to vary across voters by allowing for joint distributions, \( f(\theta^i, \xi^i, \cdot) \), over voter ideals and voter susceptibility to persuasion. Voters with extreme views, for instance, may be harder to persuade than voters with moderate views, or vice versa.

The present model can also capture and help illuminate more alternative models. As an example, Ferguson (1995) posits an “investment theory of political parties” in which parties are “analyzed as blocs of major investors who coalesce to advance candidates representing their interests.” (p. 22) In this view of the world, business elites rather than voters are the key arbiters of policy outcomes. In particular, “on all major issues affecting the vital interests that major investors have in common, no party competition will take place.” (p. 37) In terms of the present model, Ferguson’s theory can be broken down into several component hypotheses. First, rather than locating at opposite extremes, special-interest groups in fact cluster at one or the other extreme. Regardless of which party they support, special interests are business elites with essentially the same ideal policies, especially when compared with policies that would be seen as leading to favorable outcomes by the population as a whole. Second, special-interest ideal points are also candidate ideal points; indeed politicians can be considered as the agents of special-interest employers. Third, the level return to campaign spending is very high. With an extremely efficient technology for converting campaign dollars spent into votes received (for instance when all voters are basically uninformed), candidates with low campaign funds — even when they represent moderate views — are unelectable. Obviously, each of these component hypotheses is open to question. My point is that in light of the analysis presented herein, Ferguson’s conclusion that voters are relatively irrelevant to policy outcomes relies on all of the component hypotheses holding. Otherwise voter-generated centripetal forces will continue to be a major factor shaping candidate electoral behavior.

**Theoretical Extensions**

At least two broad lines of theoretical inquiry are suggested by the current model. The first involves removing the various strong assumptions that I have made. These include locating special-interest groups at the extremes of the issue space, exogenously parameterizing their
behavior, exogenously parameterizing candidate preferences, allowing candidates to commit to post-election policy positions, specifying additively separable contribution schedules and campaign technology functions, and placing restrictions on candidates’ feasible location spaces. Each assumption I would justify as allowing for an insightful first pass at understanding the determinants of candidate behavior. For a more fundamental understanding, each of the assumptions should and can be relaxed. Probably most important is the need to endogenize special-interest behavior. Without doing so, the present model is best interpreted as representing the second stage of a two-stage game; any conclusions based on comparative static analysis of exogenous shocks (for instance, the introduction of campaign finance reform laws) are surely fragile to allowing for special-interest best responses.

The second broad line of inquiry is the extension of the model into multidimensional space. As with the assumptions above, single-dimensional competition is simply too restrictive to answer certain fundamental questions. How do candidates tradeoff their positions among the various dimensions? Do they raise money in one dimension and advertise in another? What is the effect of introducing single issue voters? Only in a multidimensional framework can we begin to seek answers. The present model is ideally suited for this extension. By specifying distributions over voter ideal points that allow for correlations across dimensions, a large number of “issues” can be introduced while remaining in what is effectively a low dimensional space. By attaching voter relative strengths to each of the issues (so that utility surfaces are ellipsoid) and by allowing these to be correlated with voter ideal points, phenomena such as single-issue voting can be explored. By allowing campaign spending to affect the relative strengths voters attach to the issues, a mass election version of agenda setting can be captured.20 Finally, by attaching an orthogonal dimension on which candidate’s positions are fixed and special-interest groups absent, but on which candidates can still spend campaign funds, even “character” can be modeled. Of course, few if any analytical solutions will be available in such a multidimensional space, and failure of equilibrium will become a bigger problem. But such messiness is a feint excuse for remaining in the confining framework of a single dimension. In particular, numerical techniques such as those employed by Kollman, Miller, and Page (1992) can be used to richly characterize the nature of candidate behavior.

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20 Page (1976) argues that such an “emphasis allocation theory” can account for candidate ambiguity on divisive issues where “divisive” is modeled as a bimodal distribution over voter ideal points.
Empirical Implications

To what extent does the framework I have presented capture what is actually going on in real-world electoral competition? I have argued for a number of alternative interpretations in an effort to appeal to readers’ intuitions. What remains is first, an inductive, case-study analysis of particular candidates and particular races in light of the model’s predictions. And second, the model places systematic restrictions on candidate behavior which should be empirically falsifiable. The problem in this latter case is separating out hypotheses that intrinsically follow from the model from those that spuriously derive from simplifying assumptions. I have already argued above that the assumption of unidimensional competition obscures some fundamental electoral dynamics. A false prediction here is that a candidate who adopts an unpopular policy position (even after campaign spending) relative to that of her opponent will have a low probability of election. In fact, if the issue is one on which hardly anyone other than a few zealots attach much importance, such a position may actually strengthen a candidate’s probability of election. Similarly, the assumption that candidates’ only source of campaign funds are policy-motivated special interests obscures the myriad other reasons individuals and special interests make political donations. In particular, incumbents’ ability to help individuals and businesses on non-policy matters provide them with a source of funds unattached to any centrifugal force.

A result that I believe will survive the removal of simplifying assumptions is that candidates respond differently to changes in their opponent location depending on whether their own best response is more moderate or more extreme than their ideal policy. A possible strategy for testing this restriction is to pool time series of particular candidates’ positions on particular issues and how these vary across different elections and opponents. The key difficulty here is ascertaining candidates’ true beliefs. Levitt (1996), however, suggests a method for recovering legislator ideal points based on variations in congressional delegations’ voting patterns. Realizing such a test is a priority for future research.

5 Conclusions

By introducing special-interest contributions as a centrifugal force acting against the centripetal force towards the median voter, I have developed a framework in which to study the underlying determinants of candidate locational choice. Even purely office-oriented candidates may diverge, and policy-oriented candidates may diverge relative to their ideal policy
locations. Where any particular candidate locates depends on a complex but systematic interaction of factors including the marginal and level returns to her and her opponents divergence, her particular opponent’s location, and the candidate’s own preferences in terms of her ideal policy location and the relative weight she places on achieving policy outcomes versus the utility she receives from holding office per se.

The model yields a number of analytical and numerical results. For a purely office-oriented candidate, the locational decision is independent of her opponent’s behavior. For a purely policy-oriented candidate, an opponent’s divergence is matched by divergence when the candidate’s best response is located extreme relative to her ideal but by convergence when her best response is moderate relative to her ideal. For all candidates, increases in the level return to divergence tend to shift reaction curves towards the extremes. For candidates with a policy orientation, high marginal returns to divergence can cause reaction functions to be discontinuous; such discontinuities in turn can lead to the failure to exist of an equilibrium. Multiple equilibria are also a possibility. Nevertheless these are exceptions, and over the vast majority of the parameter space there exists a unique equilibrium.

A final point on normative content: an underlying premise of this paper is that voters have preferences not over policies but rather over outcomes. If, in fact, centrist policies correspond most closely to the policy prescriptions implied by a “true” map of how the world works, then the polarization introduced by special interests is clearly Pareto-worsening. But history suggests that “extreme” notions of how the world works have often proved more accurate than their “moderate” predecessors. Especially to the extent that variations in policies serve as natural experiments with which econometricians can better estimate any “true” underlying map, the drawing of normative conclusions is done at considerable risk.

Appendix

Rational Voter Response to Campaign Spending

The basic setup remains the same as in the model. Voters initially hold views over how the economy works, but they do so under a “veil of anonymity” regarding the distributional consequences of policies. (See Congleton and Sweetser, 1992.) Their priors thus correspond to beliefs over the distribution of benefits and costs, but voters do not know specifically how they as individuals will fare. Candidates now use their campaign funds to hire analysts who help the candidates to identify specific winners from their own committed policy and specific losers from their opponent’s policy. They also hire media consultants to help them
to inform the appropriate specific voters.

With such a modeling strategy, care must be taken in establishing a framework in which such revelation is actually in the candidates’ interests. As Fernandez and Rodrik (1989) show, the partial removal of a veil of anonymity can be counter-productive if it causes voters who do not receive specific information to revise their own beliefs. (Alternatively, then, the care should be taken by the candidates’ policy analysts and media advisers.) Nevertheless it is not too difficult to construct a setting with a similar reduced form as the present model.

For instance, for a proposed policy assume that any given voter has a $\frac{1}{2}$ probability of receiving $\epsilon$ above the policy’s mean (whatever the mean happens to be) and a $\frac{1}{2}$ probability of receiving $\epsilon$ below the policy’s mean. For each potential policy there exists an urn containing chits, one for each voter, which state whether the voter will receive $\epsilon$ more than, or $\epsilon$ less than the policy’s mean. The candidates’ policy analysts draw chits with replacement from the urns corresponding to policies $X_R$ and $X_L$. When the results are favorable to a candidate’s position (i.e. $+\epsilon$ for the candidate’s own policy, $-\epsilon$ for the candidate’s opponent’s policy) the media analysts are assumed to be able to credibly transmit this information to the appropriate voter.

The assumptions that the policy analysis contains no information on the policy’s mean and that chits are drawn with replacement help to keep voters from making strong inferences when they do not receive information. Nevertheless some revision in favor of the candidate who spends less would be warranted (i.e. non-contacted voters could rationally infer that there was a greater than $\frac{1}{2}$ probability they would receive $+\epsilon$ from the “poor” candidate’s policy and $-\epsilon$ from the “rich” candidate’s policy). The replacement assumption also helps establish decreasing returns to campaign spending. As campaign spending increases (I assume a constant cost per draw), with growing frequency policy analysts draw chits of voters for whom they already have information. Such additional spending is also likely to cause diminishing returns through the inferences being made by non-contacted voters.

**Purely Office-Seeking Candidates, Comparative Statics**

Contingent on parameters which remain within regions III and IV of Figure 1, totally differentiating the non-exponential term in (9) gives,

$$\frac{\partial X_{R,Office}}{\partial \alpha \beta_R} = X_{R,Office} \left( \ln \left( \frac{2\zeta \eta_{R\alpha \beta_R}}{1 - \alpha \beta_R} \right)^2 + \frac{1}{(1 - \alpha \beta_R) \alpha \beta_R} \right) \tag{A1}$$

> $0$ as $\zeta \eta_R > \frac{\exp \left( \frac{a \beta_R - 1}{a \beta_R} \right) \alpha \beta_R}{2 \alpha \beta_R}$

> $< 0$ as $\eta_R < \frac{\exp \left( \frac{a \beta_R - 1}{a \beta_R} \right) \alpha \beta_R}{2 \alpha \beta_R}$

31
Increases in the marginal return to divergence will cause R to diverge away from zero whenever $\zeta_\eta R$ is sufficiently large relative to $\alpha \beta_R$ as given by (A1). The partial effect of increases in level buying power on R’s locational choice can be similarly derived and show that increases in $\zeta_\eta R$ will always cause R to diverge away from zero.

$$\frac{\partial X_{R,Office}}{\partial \zeta_\eta R} = \frac{X_{R,Office}}{(1 - \alpha \beta_R) \zeta_\eta R} > 0 \text{ for } \alpha \beta_R < 1 \tag{A2}$$

**Purely Policy-Oriented Candidates, Comparative Statics of Change in One Candidate’s Level Return to Divergence**

$$\left| \frac{\partial X_R}{\partial \eta_L} \right|_{\text{symmetry}} = -\left( \frac{\sqrt{\zeta_\alpha X_R^{\alpha R}}}{\sqrt{\beta \sigma^2}} \right) \left( \sigma^2 (\theta_R - X_R) + 2\theta_R \left( \zeta_\eta R \alpha \beta_R X_R^{\alpha R} + \alpha \beta_R \sigma^2 \right) \right) - \left( \frac{\partial^2 U_R}{\partial X_R^2} \right) \tag{A3}$$

$$\left| \frac{\partial X_R}{\partial \eta_R} \right|_{\text{symmetry}} = \left( \frac{\sqrt{\zeta_\alpha X_R^{\alpha R}}}{\sqrt{\beta \sigma^2}} \right) \left( \sigma^2 (\theta_R - X_R) + 2\theta_R \zeta_\eta R \alpha \beta_R X_R^{\alpha R} \right) - \left( \frac{\partial^2 U_R}{\partial X_R^2} \right) \tag{A4}$$

For both (A3) and (A4), the denominator will always be negative at an interior equilibrium. So long as $X_R \leq \theta_R$, the numerator of (A3) is positive and the numerator of (A4), negative.
Bibliography


Hinich, Melvin J. and Michael C. Munger (1989). “Political Investment, Voter Perceptions, and


Table 1: Nature of Equilibria Outcomes

\( \theta_{LR} = \pm 0.15; \sigma = 0.2 \)

\( \alpha \beta L \times \alpha \beta R \times \zeta \eta L \times \zeta \eta R = \{0, .2, .4, .6, \ldots 2\} \)

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Figure 1: Locational Choice for Purely Office-Seeking Candidates

- **I:** Maximal Feasible Divergence ($X_R = 1$)
  - Quasiconvex in $X_R$
  - Quasiconcave in $X_R$

- **II:** Maximal Feasible Divergence ($X_R = 1$)

- **III:** Intermediate
  - $dX_R/d\alpha > 0$

- **IV:** Intermediate
  - $dX_R/d\alpha < 0$

- **V:** Maximal Feasible Convergence ($X_R = 0$)
Figure 2: Candidate Strategic Interaction

Panel A: "Moderation-Based" Strategies

\[ \alpha \beta_{LR} = 0.2; \gamma_{LR} = 0.2; \sigma = 0.2 \]

Panel B: "Money-Based" Strategies

\[ \alpha \beta_{LR} = 1; \gamma_{LR} = 0.6; \sigma = 0.2 \]

Panel C: "Money-Based" Strategies, Multiple Equilibria

\[ \alpha \beta_{LR} = 1.4; \gamma_{LR} = 1; \sigma = 0.2 \]

Panel D: Combination Strategies, Multiple Equilibria

\[ \alpha \beta_{LR} = 2; \gamma_{LR} = 1.4; \sigma = 0.2 \]
Figure 3: Changes in Marginal Return to Divergence

Panel A: Low Level Return to Divergence
Changes to both Candidates’ Marginal Returns

\[ \zeta_{\ell_R} = 0.3; \sigma = 0.2 \]
\[ \alpha \beta_{\ell_R} = a 0.1; b 0.4; c 0.95; d 1.4 \]

Panel B: Medium Level Return to Divergence
Changes to both Candidates’ Marginal Returns

\[ \zeta_{\ell_R} = 0.6; \sigma = 0.2 \]
\[ \alpha \beta_{\ell_R} = a 0.1; b 0.4; c 1.0; d 1.6 \]

Panel C: Low Level Return to Divergence
Change to only R’s Marginal Return

\[ \zeta_{\ell_R} = 0.3; \sigma = 0.2 \]
\[ \alpha \beta_{\ell_R} = a 0.1; b 0.4; c 0.95; d 1.4 \]

Panel D: Medium Level Return to Divergence
Change to only R’s Marginal Return

\[ \zeta_{\ell_R} = 0.6; \sigma = 0.2 \]
\[ \alpha \beta_{\ell_R} = a 0.1; b 0.4; c 1.0; d 1.6 \]
Figure 4: Changes in Level Return to Divergence

Panel A: Decreasing Marginal Return to Divergence
Changes to both Candidates' Level Returns

\[ \alpha \beta_{LR} = 0.8; \sigma = 0.2; \]
\[ \zeta_{LR} = a \ 0.2; \ b \ 0.3; \ c \ 0.4; \ d \ 0.6; \ e \ 1.0; \ f \ 2.0 \]

Panel B: Increasing Marginal Return to Divergence
Changes to both Candidates' Level Returns

\[ \alpha \beta_{LR} = 2.0; \sigma = 0.2; \]
\[ \zeta_{LR} = a \ 0.1; \ b \ 0.5; \ c \ 0.9; \ d \ 1.3; \ e \ 1.7; \ f \ 2.1 \]

Panel C: Decreasing Marginal Return to Divergence
Change to only R's Level Return

Panel D: Increasing Marginal Return to Divergence
Change to only R's Level Return

\[ \alpha \beta_{LR} = 0.8; \zeta_{LR} = 0.2; \sigma = 0.2 \]
\[ \zeta_{\Pi R} = a \ 0.2; \ b \ 0.3; \ c \ 0.4; \ d \ 0.6; \ e \ 1.0; \ f \ 2.0 \]

\[ \alpha \beta_{LR} = 2.0; \zeta_{LR} = 0.1; \sigma = 0.2 \]
\[ \zeta_{\Pi R} = a \ 0.1; \ b \ 0.9; \ c \ 1.7 \]
Figure 5: Changes in Candidate Preferences

Panel A: Centripetal Electoral Environment
Changes to Candidate Ideal Points

\[ \alpha \beta_{LR} = 1; \zeta_{LR} = 0.2; \sigma = 0.2 \]
\[ \theta_{LR} = a \pm 0.05; b \pm 0.15; c \pm 0.25; d \pm 0.35 \]

Panel B: Intermediate Electoral Environment
Changes to Candidate Ideal Points

\[ \alpha \beta_{LR} = 0.6; \zeta_{LR} = 0.4; \sigma = 0.2 \]
\[ \theta_{LR} = a \pm 0.05; b \pm 0.15; c \pm 0.25; d \pm 0.35 \]

Panel C: Centripetal Electoral Environment
Changes in Weight on Office Holding

\[ \alpha \beta_{LR} = 1; \zeta_{LR} = 0.49; \sigma = 0.2 \]
\[ \theta_{LR} = a 0; b 0.5; c 0.75; d 1 \]

Panel D: Centrifugal Electoral Environment
Changes in Weight on Office Holding

\[ \alpha \beta_{LR} = 1; \zeta_{LR} = 0.51; \sigma = 0.2 \]
\[ \theta_{LR} = a 0; b 0.5; c 0.75; d 0.90; e 1 \]