Self-Organized Criticality and Fluctuations in Economics

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Abstract

Frequent and sizable fluctuations in aggregate economic activity can result from many small independent perturbations in different sectors of the economy. No exogenous cataclysmic force is needed to create large catastrophic events. This behavior is demonstrated within a "toy" model of a large interactive economy. The model self-organizes to a critical state, in which the fluctuations of economic activity are given by a stable Pareto-Levy distribution.

1. Introduction

Many years ago, Mandelbrot (1960, 1963, 1964) pointed out that the variations in certain prices, employment etc. seems to follow stable Paretian distributions with large power-law tails. Paretian distributions are non-Gaussian distribution with the property that the sum of two variables with such distributions itself has the same distribution (Levy, 1924, 1954; Gnedenko and Kolmogorov, 1954). Obviously, the central limit theorem, according to which the sum must approach a Gaussian, is violated. The distribution can therefore not have any second moment, i.e there must be large tails falling off algegraically rather than exponentially. In other words, large catastrophic events occur comparatively often, seemingly in agreement with observations.

At that time, no indication of the mechanisms which could lead to this type of behavior was given, and none has surfaced since. Recently, however, it was pointed out that large interactive dissipative dynamical systems typically self-organize into a critical state, far out of equilibrium, with fluctuations of all sizes (Bak, Tang, and Wiesenfeld, 1988; Bak and Chen, 1991) Dissipative systems are open ones, where energy, mass, etc. is continuously fed in and eventually burned ("dissipated"). The discovery has a;lready provided insight into a variety of phenomena in physics, such as earthquakes (Bak and Tang, 1989), volcanic eruption (Diodati et al., 1991), turbulence, and even Biology (Bak, Chen, and Creutz, 1991; Kauffman and Johnson, 1991, and political upheaval (Schrodt, 1991), where large events with Paretian-like distributions had also been observed, but never understood. Can these ideas be applied to describe economiiess, which might indeed be viewed as "large interactive dissipative" dynamical system"? As a first step towards addressing this question, we have analysed a simple model of an interactive production economy, which indeed self-organizes in to a critical state with large fluctuation in aggregate production following a stable Pareto-Levy law.

Instability of economic aggregates is a well known puzzle. A number of possible reasons for variation in the pace of production can be mentioned: stochastic variation in timing of desired consumption of produced goods, or stochastic variation in production cost. But it is hard to see why there should large variations in these factors that are synchronized across the entire economy - in stead it seems more reasonable to suppose that variations in different people's demand for different produced goods occur independently: so, why should one not expect the variations in demand to cancel out in the aggregate, by the law of large numbers leading to

Gaussian distributions with harmless exponential tails? Similarly, stochastic variations in production costs should be independent across sectors - so why shouldn't they cancel out in their effects on aggregate production?

The conventional response is that aggregate shocks are needed as the source of business cycles, i. e. large cataclysmic forces that affect the entire economy in a similar way. Especially important candidates are changes in government policy that affect financial markets, and through them the entire economy, or that affect the budgets of many people in the economy at once.

An alternative scenario which has recently been suggested is that model economies possess some intrinsic deterministic dynamics, which (even in the absence of external shocks) involve persistent fluctuations, such as a limit cycle, or even chaotic behavior related to a strange attractor. The problem is that these models that aggregate fluctuations should involve motion on a low-dimensional attractor. Yet, analysis of economic time series hasn't found evidence of this. The chaotic motion is essentially white uncorrelated noise, which is very different from the Pareto-type behavior which has been observed.

The alternative we wish to pursue is that the effects many small independent shocks to different sectors of the economy don't cancel out in the aggregate due to a failure of the law of large numbers. This can occur as a result of nonlinear, local interactions between different parts of the economy. We have constructed a simple model of interacting economic agents, or sectors. The agents buy, produce, and sell goods to neighbor sectors in the economy. The economy is driven by small, independent random shocks of demand, but nevertheless the resulting aggregate production converges to a stationary Pareto-Levy distribution. The Pareto-Levy law for this particular model has the property that its mean value and its width scales with the number of shocks, N, with the same exponent, in contrast to a Gaussian distribution where the width scales with $N^{1/2}$ and the mean increases as N. Thus, the magnitude of the relative fluctuations do not decay at all as the number of shocks become large, as long as the total number of shocks is small compared with the total number of interacting agents in the economy. This result is obtained by utilizing a mapping of the model onto an exactly solvable model of self-organized criticality constructed by Dhar and Ramaswamy (1989). But first a brief interlude on self-organized criticality.

2. On self-organized criticality.

The phenomenon of self-organized criticality appears to be quite universal and provides a unifying concept for large scale behavior in systems with many degrees of freedom. It complements the concept of chaos wherein simple systems with a small number of degrees of freedom can display quite complex behavior.

The prototypical example of self-organized criticality is a sand pile. Adding sand to, or tilting, an existing heap will result in the slopes increasing to a critical value where an additional grain will give rise to unpredictable behavior. If the slopes were too steep, one would obtain one large avalanche and a collapse to a flatter and more stable configuration. On the other hand, if it were less steep the new sand will just accumulate to make the pile steeper. The critical state is an attractor of the dynamics. At the critical slope the distribution of avalanches has no overall scale (there is no typical size of an avalanche). The large fluctuations provide a feedback fast mechanism returning the slope to the critical one, whenever the system is modified, for instance by using different types of sand. The most spectacular achievement is probably the explanation of the famous Gutenberg-Richter law (1956) for the distribution of earthquakes.

Self-organized criticality seems to go against the grain of classical mechanics. According to that picture of the world, things evolve towards the most stable state, i. e. the state with lowest energy. So all that classical mechanics says about the sand-pile is that it would be more stable if spread out in a flat layer. That is true, but utterly unhelpful for understanding real sand-piles. The most important property of the self-organized criticality is its robustness with respect to noise and any modifications of the system: this is necessary for the concept to have any chance of being relevant for realistic complicated systems.

3. The model.

We now argue that we need both local interactions (each sector or agent interacts only with a small number of other agents or sectors) and significant nonlinearity of responses to variations in demands in order to get non-trivial aggregate behavior.

In the standard macro model the entire economy is a single market in which the aggregate of all producers use their joint production capacity (represented by an "aggregate production function") to supply the aggregate quantity demanded by all buyers. Production possibilities involve diminishing returns (convex cost functions). The result is that the aggregate demand for produced goods should be relatively smooth. If there are N buyers, each of whose purchases are independent drawings from a distribution with mean m and variance σ , the ratio of the standard deviation to the mean of the aggregate demand decreases as $N^{-1/2}$ as N becomes large. Furthermore, aggregate production should be even smoother than aggregate sales. With a convex cost function the producer can minimize costs by smoothing production (through inventory variations) even if there is some cost of inventory variations.

Local interaction doesn't change much as long as we continue to assume convex costs. Suppose there are many productive units, each of which supplies only a small number of customers, e. g. a grid as shown in figure 1.

Assume a finite cylindrical grid of height and length L, and consider the effects of letting dimensions become large. Now, each final goods producer faces significant fluctuations in demand (standard deviation/mean is of order unity, not $N^{-1/2}$) but average production by all final goods producers still has low s.d./mean, because their individual fluctuations are independent.

The demand faced by intermediate goods (i.g.) producers, far back in the production chain, is an average of demand for final goods in many different sectors; this becomes progressively more correlated, the further back one goes in the chain. But if the producer's production response to sales is a linear function, then each i.g. producer's demand is an average of independent random variables, which has progressively lower s.d/mean further back in the chain. So there is no great aggregate variability of production at any level of the grid, if L is large. Approximate linearity arises near the optimal production level for convex cost function.

Non-convexities also don't change much as long as we assume an aggregate model. Non-convex costs can arise because of indivisibility: one has to either operate an assembly line or close it down altogether. The resulting cost function may look something like this:

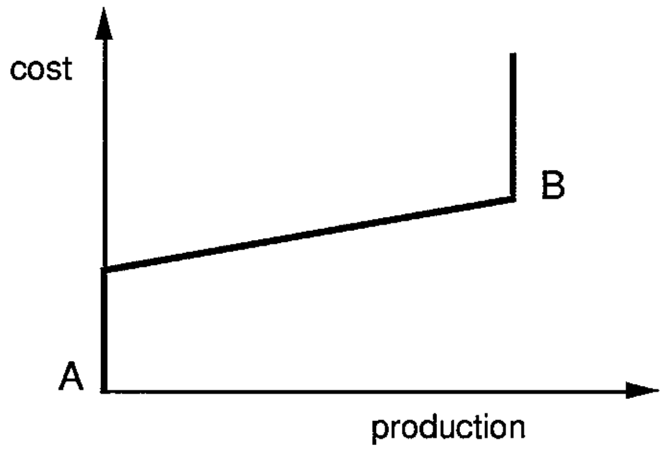

Fig. 2. Non-convex cost function for indivisible production facility.

In such a case costs are minimized by alternating between production at points A or B, if average level of sales is in between. This can obviously lead to fluctuations in production that are larger than the fluctuations in sales; we could even have fluctuating production with steady sales flow (periodic bursts of production to replenish inventories).

Non-convexities don't change much either, as long as we assume an aggregate model. But this can only explain aggregate fluctuations if the individual indivisibilities are large compared to the economy as a whole. The standard argument is that while the productive sector of an economy might well be made up of many small productive units, each of which may have a non-convex technology of the kind above, the aggregate production possibilities are well approached by a convex function. For a macroscopic shift of aggregate sales, the effect on the aggregate production is still approximately as in the convex model.

Thus, in order to have large aggregate fluctuations beyond the Gaussian ones emerging from the central limit theorem, we need both locality and non-convexity, both of which might well be relevant in a real economy.

In our model, agents are defined on the cylindrical lattice of figure 1, with coordinates (i,j), where i = 1,2,....,L; j = 1,2,...,W, i being the column number, j the row. We use modulo L arithmetic for the horizontal direction, i. e. the lattice is a cylinder. There are a total of $M=L*W$ agents. Each agent buys goods from two suppliers, uses these goods to manufacture his own goods, and sells to two customers. Each agent can hold an inventory of 0 or 1 units of his own goods.

The initial state of the economy at the beginning of any period (at time t=1,2,....) is described by specification of the inventory holdings $x_{i,j}(t)$ for each firm, with x = 0 or 1. There are thus 2^M possible states in the configuration space for this economy.

The economy is activated by random orders at time t received by final goods producers (firms in row i=1). Each random order initiates a chain reaction. If the firm receiving the order can fill the order out of already existing inventories ($x_{1,j}(t)=1$) , it does so and the inventory vanishes, $x_{1,j}(t) \to 0$. Otherwise it decides to produce $y_{1,j}(t) = 2$ units of output. In order to do so, it must order 1 unit of supplies from each of its suppliers (2,j) and (2,j-1), thus $S_{2,j}(t) = S_{2,j-1}(t) = 1$. The second unit not sold to (1,j) is placed in inventory, $x_{1,j}(t) \to 1$. The orders $S_{2,j}(t)$ may trigger production at firms in row 3 following the same rules.

The chain reaction stops whenever there are no more need for production, i.e. $x_{1,j}(t)=1$ for all firms receiving orders, or when i=L and the firm is a primary materials producer. The process is repeated at time t+1, starting with the final configuration at time t, and so on.

To see these rules in action, consider the configuration in figure 3 (top) where black dots indicate an inventory of 1 unit, the white circle an inventory of zero units. The agent at site (1,4), at the arrow, recieves an order at time $t = t_0$. Since he has nothing in stock he must order one units from each of the firms at sites (2,3) and (2,4). None of these agents can deliver immediately, and have to order from the 3 agents below. Eventually the chain reaction stops in the 7th row. A total of 8 firms had to start production in order to fill the orders, so the production stemming from the single initial shock was 16, i. e. $Y(t_0)=16$. The finil configuration is shown at the bottom of figure 3. This particular avalanche did not reach the bottom row of primary materials suppliers so there is an aggregate loss of 1 unit from the system.

We monitor the aggregate production $Y(t) = \Sigma_{i,j} y_{i,j}(t)$ as time progresses. We are interested in the kind of fluctuations in Y(t) that results from many independent shocks to the sales of final goods suppliers $S_{1,j}(t)$. Can we have the number of independent shocks $S_{1,j}$ large, but nevertheless be left with significant variability of aggregate production? Fortunately, the model is isimorphous with a model of Self-organized Criticality for which most of the preperties have been derived rigorously by Dhar and Ramaswamy (1989), so we don't have to; it is essentially the only exactly solvable model of Self-organized Criticality so far. Usually one must resort to numerical methods in order to derive the properties of models of SOC.

Dhar and Ramaswamy found that starting from scratch (zero inventory) the model evolves to a statistically stationary critical state. Shocks S will lead to avalanches penetrating through r rows and involving a total production Y. The distribution of r is

$$P(r) = (2r)!/[r!(r+1)!]\ 2^{-2r-1} \tag{1a}$$

which has the asymptotic limit

$$P(r) = r^{-3/2}, \tag{1b}$$

for large r, but $r < L$. There is an excess "bump" at $r=L$ stemming from the cutoff of larger avalanches, $P(r>L) \approx L^{-1/2}$. Similarly, the asymptotic distribution of the aggregate production Y, for $Y>>1$, is

$$P(Y) \approx Y^{-1-\tau},\ \tau = 1/3 \tag{2}$$

again with a (this time smoother) cut-off around $Y = Y_{cutoff} \approx L^{3/2}$ assuring that the average production following an initial order is L. Again, there is an excess probability centered around the cutoff stemming from larger avalanches halted at $r=L$, given by $P(r=L,Y)$. The integrated probability contained in that peak decays as $L^{-1/2}$. Thus, the peak is a finite size effect vanishing as $L \to \infty$. The model can easily be generalized to any dimension d, with $\tau = 3/2$ for $d = 3,4....$ Since the lattice is somewhat artificial for the economy anyway, there is no reason to believe that the $d=2$ result is more relevant than the $d > 3$ result.

We now consider the effect of having many shocks simultaneously. One might worry that the fact that the avalanches caused by two independent shocks might overlap, in particular for a large probability p, would affect the joint probability distribution function $P_2(Y)$ for the production caused by the two shocks. Fortunately, for this particular model, this is not the case. One can easily calculate the distribution $P_2(Y)$ for two nearest neighbor shocks rigorously. A single initial order leads to production with probability 1/2. The second row is then subjected to two neighbor shocks: this gives a simple recursion relation for P(x):

$$P(x) = 1/2\ P_2(x-1),\ \text{i.e}\ P_2(x)=2P(x+1).$$

For large x, $P(x) \approx P(x-1)$, so $P_2(x) = 2\ P(x+1)$, exactly as it would have been if the events were uncorrelated. In general, for n successive initial sales $P_n(x)=nP(x)$ as long as x>>n. In the following, only the asymptotic behavior is important so correlation effects are negligible.

The model is "Abelian" in the sence that the final aggragate production of two shocks does not depend on the sequence of those shocks (although the individual magnitudes do), or whether the shocks occur simultaneously or separately. This is easily seen by noting that the production at a given site depends only on the total number of orders recieved up to and including the present time, independently of how those orders were recieved. More generally, the effect of many shocks simultaneously is the same as the effect of many consequtive shocks, so only the total amount of shocks, N, is important. In more general models of SOC there are correlation effects complicating substantially the analysis.

Thus, we study the distribution of the random variable z defined as the sum of N independent, random variables

$$z = \sum_{i=1}^{N} y_i \tag{3}$$

where $P(y_i) = y_i^{-1-\tau}$. We replace the discrete variable y with the corresponding continuous variable y, y>1, without loss of accuracy for large y, and, as we shall see, without affecting the asymptotic limit for large z. The distribution function for z can then be represented by an integral rather than a sum:

$$Q(z) \approx \int dy_1 dy_2 \cdots dy_{N-1} P(y_1)P(y_2) \cdots P(y_{N-1})P(z - y_1 - \cdots - y_{N-1}) \tag{4}$$

This distribution function is found by introducing the Fourier transforms

$$\tilde{P}(k) = \int e^{iky} P(y)dy$$

$$\tilde{Q}(p) = \int e^{ipz} Q(z)dz \tag{5}$$

and their inverse

$$P(y) = (1/2\pi) \int e^{-iky} \tilde{P}(k)dk$$

$$Q(z) = (1/2\pi) \int e^{-ipz} \tilde{Q}(p)dp \tag{6}$$

We are interested only in the behavior for small k, since only this will affect the equation for large z. One finds, by inserting eqs. (6) t into eq. 2, a simple equation relating the Fourier transforms of the original avalanches with the Fourier transform of the distribution of the sum of avalanches:

$$\tilde{Q}(p) = [\tilde{P}(p)]^N \tag{7}$$

The simple product form is due to the independence of the random variables.

We first find P(k). The asymptotic power-law form of P(y) is sufficient in the large N, and thus small k, limit:

$$\begin{aligned}
\tilde{P}(k) &= \int_0^\infty e^{ikx} x^{-1-\tau} dx \\
&= \int_0^\infty \cos(kx) x^{-1-\tau} dx + i \int_0^\infty \sin(kx) x^{-1-\tau} dx \\
&= 1 - \int_0^\infty (1-\cos(kx)) x^{-1-\tau} dx + i \int_0^\infty \sin(kx) x^{-1-\tau} dx \\
&= 1 - k^\tau \int_0^\infty (1-\cos(y)) y^{-1-\tau} dy + i \int_0^\infty \sin(ky) y^{-1-\tau} dy \\
&= 1 - k^\tau \Gamma(\tau)\cos(\tau\pi/2) + i\, k^\tau \Gamma(\tau) \mathrm{sign}(k) \sin(\tau\pi/2)] \\
&= 1 - \gamma k^\tau [1 + i\ \mathrm{sign}(k)\tan(\tau\pi/2)]
\end{aligned} \tag{8}$$

where the constant γ is defined by the last equation. $\Gamma(x)$ is the "gamma" function. The equations are valid for $0 < \tau < 1$, that is in the case where the distribution of events has neither a second moment (the central limit theorem does not apply), nor an average). For small k and large N we find

$$\begin{aligned}
\tilde{Q}(k) &= (1 - \gamma k^\tau [1 + i\,\mathrm{sign}(k) \tan(\tau\pi/2)])^N \\
&= e^{-N\gamma k^\tau [1 + i\,\mathrm{sign}(k)\tan(\tau\pi/2)]}
\end{aligned} \tag{9}$$

and thus

$$\begin{aligned}
Q(y) &= (1/2\pi) \int e^{-iky - N\gamma k^\tau [1 + i\,\mathrm{sign}(k)\tan(\tau\pi/2)]} dk \\
&= 1/N^{1/\tau} G(y/N^{1/\tau})
\end{aligned} \tag{10}$$

where G(x) is precisely a stable Paretian distribution (see for instance Mandelbrot 1963, or Gnevdenko and Kolmogorov, 1954)

$$G(x) = (1/2\pi)\int e^{-iky - \gamma k^{\tau}[1 + i\,\mathrm{sign}(k)\tan(\tau\pi/2)]}\,dk \tag{11}$$

with $\alpha = \tau$, β=1, δ =0 in Mandelbrot's notation. The latter equation is derived from the substitution $k=k'/N^{1/\tau}$ in the integral. The skewness parameter β is non-zero because our avalanches are positive definite; the distribution can not extend to negative values; this is assured by β=1. Equation 10 with G given by the scaling function (11) is our fundamental result. The sum of avalanches approaches an invariant distribution function, whose width scales as $N^{1/\tau}$ rather than as $N^{1/2}$ as in the Gaussian case. This result applies for all self-organized critical systems with uncorrelated avalanches with $0 < \tau < 2$. For $0 < \tau < 1$ the mean value itself scales as $N^{1/\tau}$ in contrast to the Gaussian case where the mean goes as N. The relative magnitude of the fluctuations do not decay at all as the number of shocks increases. For $1 < \tau < 2$ the width scales as N as in the Gaussian case, so width/mean decays as $N^{1/\tau - 1}$, i. e. slower than in the Gaussian case. We have thus demonstrated how such a distribution naturally arises from the dynamics of a simple economy. To our knowledge, there is no prior attempt to explain the ubiquitous Pareto-Levy distribution directly from a model economy.

There exists no closed analytical expression for the function G. In order to demonstrate the convergence and the scaling, we have numerically calculated the sum y of N random drawings with the power-law distribution function, with N= 10 and N = 50, and estimated the distribution function by repeating the procedure 1 million times. The distribution P(x) was plotted versus the rescaled coordinates $x = y/N^{1/\tau}$. Figure 4 shows the result for τ = 1/2, which applies to our model for d > 2. The distribution appears to approach a unique function (as it must), with the proper scaling. The asymptotic behavior for large x is

$$G(x) \approx \sin(\pi\tau/2)\,\Gamma(1+\tau)\,/\,x^{1+\tau} \tag{12}$$

The function G decays like the probability of the individual avalanches. The significance of this is that the "catastrophic" large events in the tail are dominated by single avalanches in the original model, in contrast to the Gaussian case where the exponential tail originates from many events accidentally conspiring. This has the psychological effect that we tend to associate the large events with unique identifiable events not belonging to the "correct" statistical distribution: we throw the large events away, and "detrend" the data before analysing them. The futility of this approach was thoroughly discussed by Mandelbrot (1963).

In the mathematical analysis it was assumed that the system is infinitely large. If the production chain has a finite depth L, the size of the avalanches will be cut-off at $L^{3/2}$. The typical largest event (the median), y_m arising from N shocks in a system of size L is given by the integral

$$N \int_{y_m}^{\infty} P(y) \approx 1\,,\; P(y) = y^{-1-\tau} \Rightarrow$$
$$N \approx y_m^{\tau} \approx L^{3\tau/2} = L^{1/2} \tag{13}$$

Thus, if the number of individual events (times the number of periods over which we are summing) greatly exceeds $L^{1/2}$. , the distribution will eventually converge to a gaussian with mean NL and width $L\sqrt{N}$. The size of the largest catastrophic events are naturally limited by the size of the economy. If we progressively increase the interval over which we are measuring the production, from days, to weeks, to months, to years, we may eventually observe Gaussian fluctuations, but with a width depending on the system size.

Similarly, there is a limitation on N due to the width W of the system. The "random walks" must not interfere with the boundaries. This will be fulfilled as long as the lengths r of all the N walks are likely to be less than $L_W = W^2$. Using arguments as above, this requires $N < \sqrt{L_W} = W$. Thus, to summarize, the distribution converges to the Paretian one if i) $p=N/W<<1$; $N<<\sqrt{L}$.

In closing, we would like to point out that while the specific model studied here is definitely grossly oversimplified, and only meant to illustrate a general principle. Power-laws, and consequently stable Pareto-Levy lawss are fingerprints of an underlying cooperative, critical dynamics, and no low-dimensional dynamical picture has been able to explain the Paretian distribution. It is known from physics that low-dimensional models are generally unable to give power-law distributions. Here we have focussed on production, but since prices, production, etc are all coupled, avalanches of all these variables go hand in hand, and should show similar power laws. More work on more relevant models should be performed, and more analysis of real economic data would be helpful.

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Figure Captions

Figure 1. Grid of interacting producers and consumers. The economy is driven by demand by consumers of final goods in the upper row. Each producer supplies only a small number of customers, as indicated by arrows.

Figure 2. Non-convex cost function for indivisible production facility.

Figure 3. Avalanche triggered by a single shock at the arrow. Black dots: one unit in inventory; white dots: 0 units in inventory. Gray dots: production necessary to fill orders.

Figure 4. Distribution of 1 million sums of N avalanches with distribution $P(y) = y^{-1/2}$. The distribution converges to the Pareto-Levy distribution with $\alpha=1/2$, $\beta=1$.

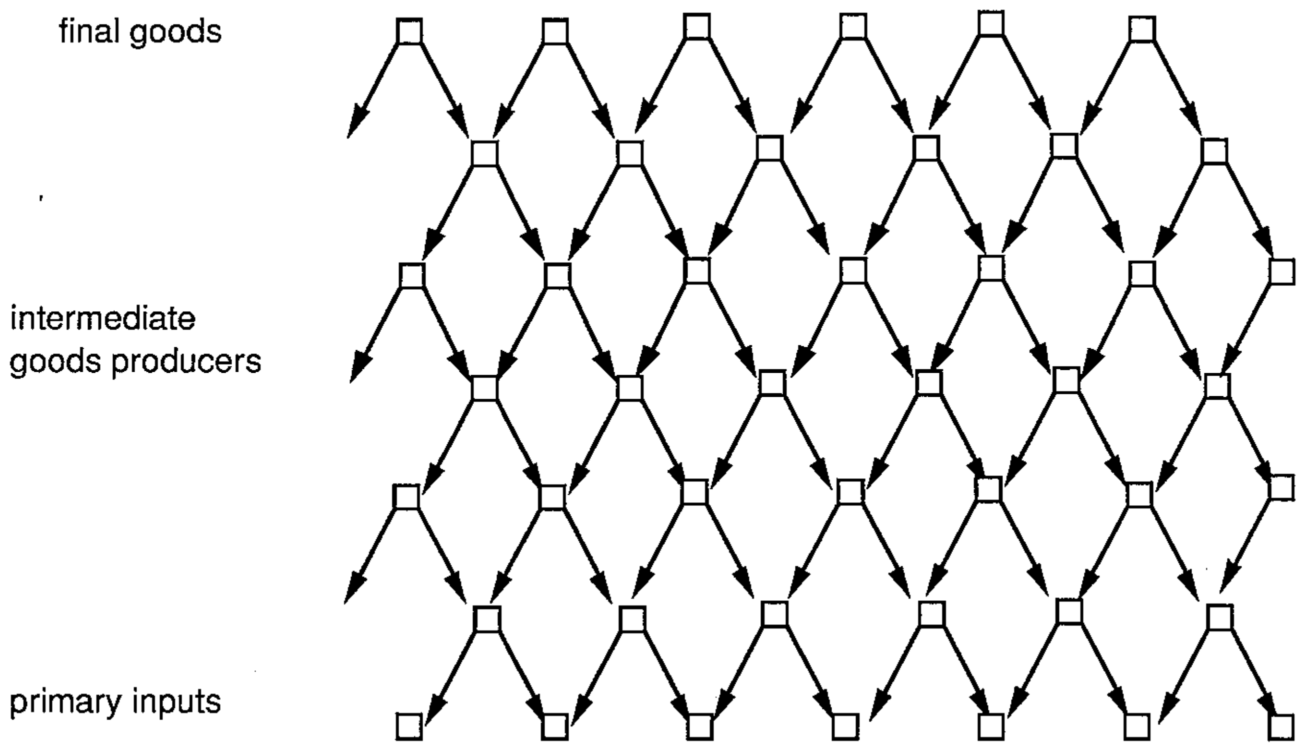

Fig. 1

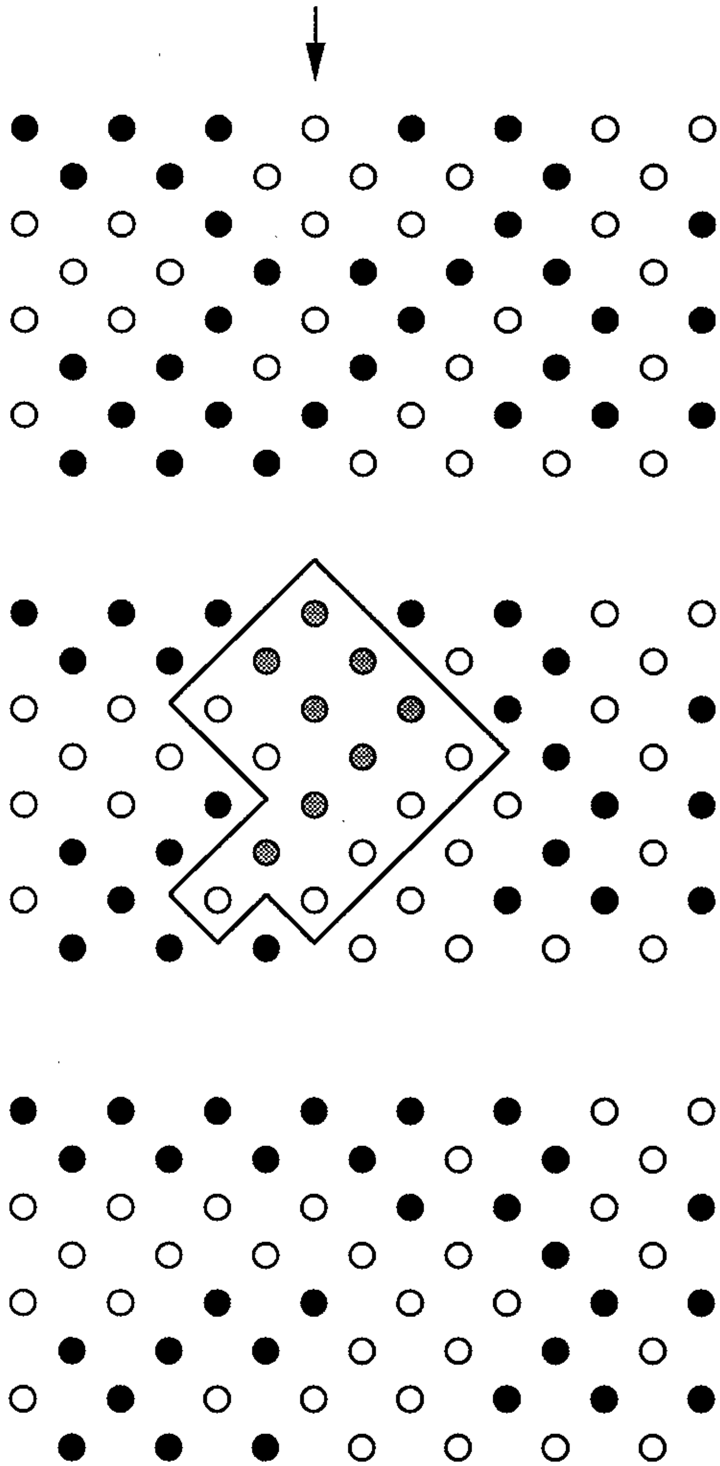

Fig. 3

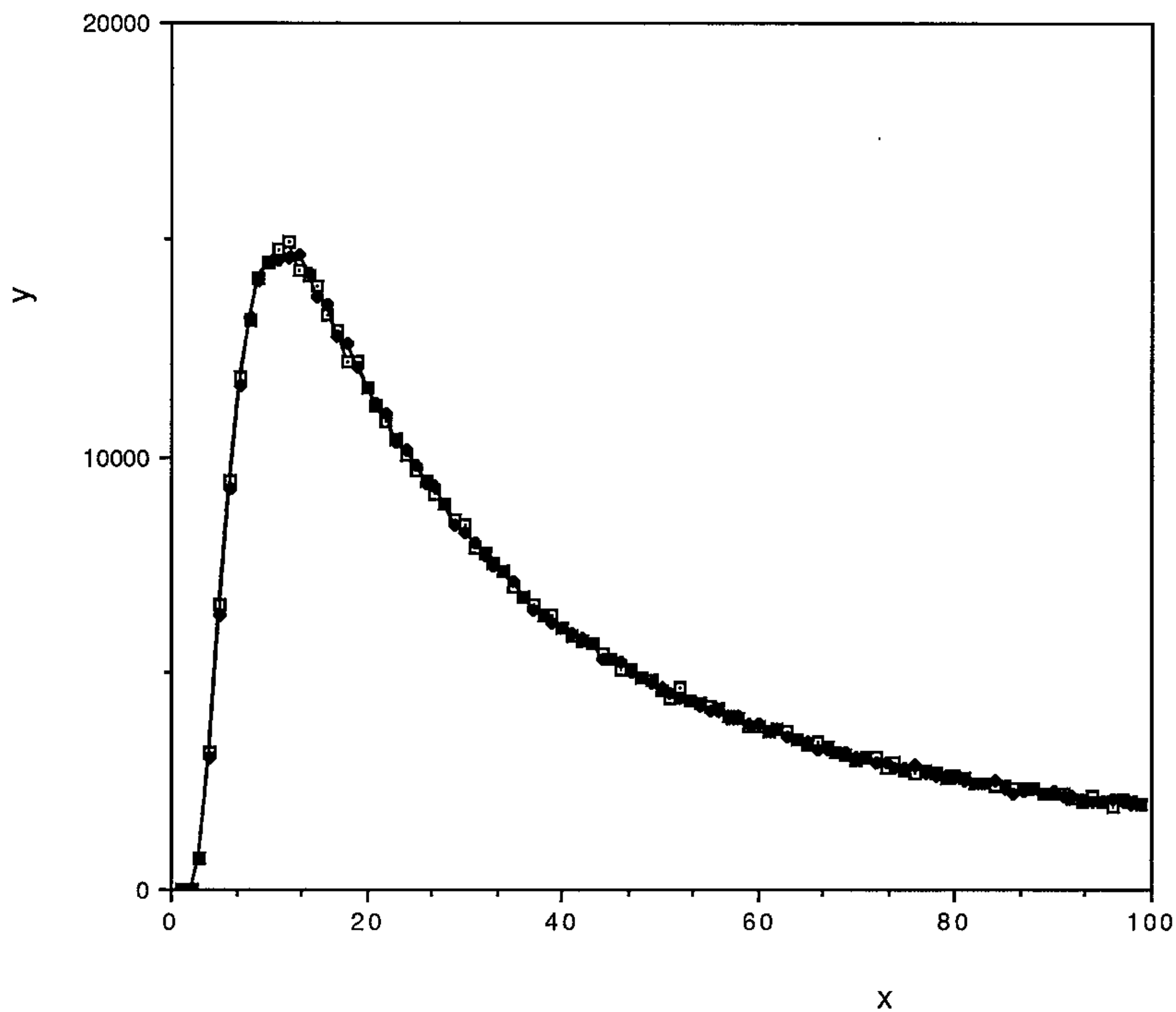

Fig. 4