Nature Conformable to Herself

What follows is a talk given by Murray Gell-Mann at the 1992 Complex Systems Winter School

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More than thirty years ago, I was the first visiting professor at the College de France in Paris, with an office in the laboratory of experimental physics. I noticed that my experimental colleagues were frequently drawing little pictures in their notebooks, which I assumed were diagrams of experimental apparatus. But it turned out that those drawings were mostly of a gallows for hanging the vice-director of the lab, whose rigid ideas drove them crazy.

I got to know the sous-directeur, and talked with him on various subjects, one of which was Project Ozma, the attempt to detect signals from another technical civilization on a planet of a nearby star. SETI, the Search for Extraterrestrial Intelligence, is the present-day successor of that project. "How could you communicate if you found such a civilization?" he asked, assuming both interlocutors would have the patience to wait for the signals to be transmitted back and forth. I suggested that we might try beep, beep-beep, beep-beep-beep, for 1, 2, 3, and so forth, and then perhaps 1, 2, 3,..., 92 for the stable (except 41 and 63) chemical elements, etc., etc. "Wait," said the sous-directeur, "That is absurd. The number 92 would mean nothing to them...why, if they have 92 chemical elements, then they must also have the Eiffel Tower and Brigitte Bardot."

That is how I became acquainted with the fact that French schools taught a kind of neo-Kantian philosophy, in which the laws of nature are nothing but Kantian "categories" used by the *human mind* to grasp reality. [Many also taught, by the way, that artistic criticism is absolute and not a matter of taste, while the opinion that artistic standards are relative was treated as a feature of Anglo-Saxon pragmatism.]

Another notion of a quite different kind, far more Platonic, is rife in mathematical circles in France (and elsewhere). That is the idea that the structures and objects of mathematics, say Lie groups, have a reality, that they exist in a sense, somewhere beyond space and time. [It is easy to see how one can come to think that way. Start with the positive integers—they certainly exist, in the sense of being used to count things. Number theory —OK. Zero and negative numbers—why not? Fractions, square roots? Solutions of algebraic equations in complex numbers? Probably—one is on a slippery slope.]

These two points of view are argued in a book, *Matière à Pensée*, published recently by the biologist Jean-Pierre Changeux and the mathematician Alain Connes. I shall not inflict all their philosophical arguments on this congenial group, and anyway I have never studied them carefully. Let me say merely that the authors do raise the question of what is the role of mathematical theory in our understanding of the world, especially the physical world.

I like to put the relevant questions in the following form: Would advanced complex adaptive systems on another planet come up with anything like our mathematics or anything like our mathematical theories of physical processes, or both? At present, we can only speculate about the answers, but the questions are deep and meaningful.

Eugene Wigner once wrote an article entitled, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." I don't know what he wrote in the article, but it is certainly a fact that up to now, especially in the domain of fundamental physics, we have had striking success with our use of mathematics.

Sometimes, as with Fourier series, the physicist has to invent the mathematical trick and the mathematicians later formalize and adapt it. Sometimes, as with Heisenberg and matrices, the concept is already known to mathematicians and physicists, but not to the particular theoretician involved, he re-invents it. Often, as with Einstein, the physicist senses what he wants and asks a mathematician to provide it—in the case of the equation describing general relativistic gravitation, Einstein asked his old classmate, Marcel Grossmann, for the tensor he needed, and thus the Ricci tensor became the Ricci-Einstein tensor.

More recently, abstract mathematics reached out in so many directions and became so seemingly abstruse that it appeared to have left physics far behind, so that among all the new structures being explored by mathematicians, the fraction that would even be of any interest to science would be so small as not to make it worth the time of a scientist to study them.

But all that has changed in the last decade or two. It has turned out that the apparent divergence of pure mathematics from science was partly an illusion produced by the obscurantist, ultra-rigorous language used by mathematicians, especially those of a Bourbakian persuasion, and by their reluctance to write up non-trivial examples in explicit detail. When demystified, large chunks of modern mathematics turn out to be connected with physics and other sciences, and these chunks are mostly in or near the most prestigious parts of mathematics, such as differential topology, where geometry, analysis, and algebra come together. Pure mathematics and science are finally being reunited and, mercifully, the Bourbaki plague is dying out. (In the late Soviet Union, they never succumbed to it in the first place.)

An anecdote will illustrate the situation during the '50s. In 1955, at the Institute for Advanced Study in

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Princeton, Frank Yang was discussing with other physicists the recently developed Yang-Mills quantum field theory. At the same time, S. S. Chern was lecturing on pure mathematics. Not only did Frank attend some of the lectures, but he and Chern were old friends, their children played together, and Chern had been one of Frank's teachers in China; neither of them noticed that Chern's lectures on fiber bundles were basically concerned with the same subject as Frank's lectures on Yang-Mills theory! In fact, they didn't learn about this equivalence for many years.

Yang-Mills theory, from the physics point of view, was a generalization of quantum electrodynamics from the gauge group U_1 to the gauge group SU_2 with non-commuting charges. Later, we generalized it to all products of U_1 factors and simple compact Lie groups, including SU_3. Today the "standard model" of elementary particle physics, apart from gravitation, is based on $SU_3 \times SU_2 \times U_1$. Moreover, Einsteinian gravitation has strong parallels with generalized Yang-Mills theory, although the gravitation theory is based on the non-compact Lorentz group and involves a tensor instead of a vector field. What does it mean that this progression from one gauge group to another has worked so well? Are we really dealing with something peculiar to the human mind or with a phenomenon so deeply rooted in the properties of nature that any advanced complex adaptive system would be likely to follow similar paths?

A related set of issues was discussed more than three hundred years ago, especially by Isaac Newton. Children learn that he thought of the theory of universal gravitation when an apple fell on his head. Well, not on his head, but nearby, anyway.

Historians of science are not sure whether to credit the apple at all, but they admit that there could have been an apple. As you know, in 1665 the University of Cambridge closed up on account of the plague and sent everyone home, including Newton, a fresh B.A., who went back to Woolsthorpe, Lincolnshire. There, during 1665 and 1666, he thought a little about integration and differentiation, a little more about the law of gravitation, and a lot about the laws of motion. Moreover, he carried out the experiment showing that white light is made up of the colors of the rainbow. While historians of science now emphasize that he didn't completely clear up all these matters in one "annus mirabilis," or "marvelous year," they admit that he made a good start on all of them around this time. As my friend Marcia Southwick says, he could have written a pretty impressive essay on "What I Did During My Summer Vacation."

As to the apple, there are four independent sources. One of them, Conduitt, writes:

> "In the year 1666 he retired again from Cambridge...to his mother in Lincolnshire & whilst he was musing in a garden it came into his thought that the power of gravity (w^{ch} brought an apple from the tree to the ground) was not limited to a certain distance from the earth but that this power must extend much farther than was usually thought. Why not as high as the moon said he to himself & if so that must influence her motion & perhaps retain her in orbit, whereupon he fell a calculating what would be the effect of that supposition but being absent from books & taking the common estimate in use among Geographers & our seamen before Norwood had measured the earth, that 60 English miles were contained in one degree of latitude on the surface of the Earth his computation did not agree with his theory & inclined him then to entertain a notion that together with the force of gravity there might be a mixture of that force w^{ch} the moon would have if it was carried along in a vortex...."

What interests us here is the extrapolation—if gravitation applies on earth, why not extend it to the heavens and use it to explain the force that keeps the moon in its orbit? Here is how Newton describes the idea much later:

> "How the great bodies of the earth Sun moon & Planets gravitate towards one another what are the laws & quantities of their gravitating forces at all distances from them & how all the motions of those bodies are regulated by those their gravities I shewed in my Mathematical Principles of Philosophy to the satisfaction of my readers: And if Nature be most simple & fully consonant to her self she observes the same method in regulating the motions of smaller bodies which she doth in regulating those of the greater. This principle of nature being very remote from the conceptions of Philosophers I forbore to describe it in that Book least I should be accounted an extravagant freak & so prejudice my Readers against all those things which were the main designe of the Book."

Today some of us have the same concerns about extrapolation as the ones to which Newton refers—Shelly Glashow inveighs against superstring theory because in embracing Einsteinian gravitation, along with the other forces, it achieves its synthesis around the Planck mass of 2 x 10^{19} GeV, larger by a gigantic factor than any energy at which particle physics experiments are carried out. But he and others were in the forefront of extrapolating the standard model SU_3 x SU_2 x U_1 to a unified Yang-Mills theory based on SU_5, in which the unification (without gravitation) is achieved around 10^{14} or 10^{15} GeV, which lies most of the way to the Planck mass. Moreover, Shelly and others have alleged that nothing much could happen in between present energies and 10^{14} GeV or so—there would just be a desert.

Well, here in Arizona we know that deserts are not necessarily empty, and that they are often very rich in plant and animal life, so the gap between our experimental energies of a few hundred GeV and 10^{14} GeV may well contain some interesting flora and fauna, especially the supersymmetric partners of the known particles—as suggested by superstring theory.

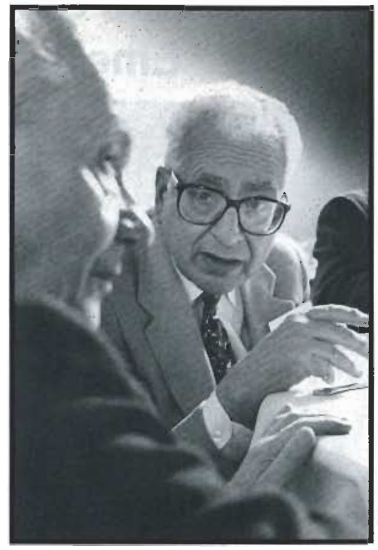
In fact, the unified super-Yang-Mills extrapolation works much better than the straight unified Yang-Mills theory (what some people call, quite inappropriately in my opinion, "grand unified theory").

But, to return to Newton, he was not thinking only of extrapolation. He returns repeatedly in his writings to the idea that Nature is consonant and conformable to herself in more general ways. From the Opticks:

> "For Nature is very consonant and conformable to her self...For we must learn from the Phaenomena of Nature what Bodies attract one another, and what are the Laws and Properties of the Attraction, before we enquire the Cause by which the Attraction is perform'd. The Attractions of Gravity, Magnetism, and Electricity, reach to very sensible distances, and so have been observed by vulgar Eyes, and there may be others which reach to so small distances as hitherto escape Observation; and perhaps electrical Attraction may reach to such small distances, even without being excited by Friction."

Thus he thought of the laws as exhibiting conformability among themselves as well as within each one, just the kind of idea that we have followed in going from electrodynamics to QCD and the electroweak theory and then onward to unified Yang-Mills theory and, with gravitation included, to the superstring theory.

If we modernize Newton's conception a bit, we could say that the laws of Nature exhibit a certain amount of self-similarity and not, of course, perfect scaling, but rather the kind of thing one sees in the Mandelbrot fractal set. So, in peeling the skins off the onion of fundamental physics, we encounter certain similarities between one layer and the next. As a result, the mathematics with which we become familiar on account of its usefulness in describing one layer suggests new mathematics, some of which may apply at the next layer—in fact even the old mathematics may still be useful at the next layer. These generalizations may be performed either by theoretical physicists or by mathematicians. If pure mathematicians are exploring ambitious generalizations of known mathematical structures, they will surely run across some of the new ones that are needed—along with much more besides.



Murray Gell-Mann and SFI Science Board Member Manfred Eigen, Max Planck Institute, at the recent Science Symposium.

Ultimately, then, we can argue that it is the self-similarity of the structure of fundamental physical law that dictates the continuing usefulness of mathematics. Suppose that the fundamental theory of the elementary particles and their interactions is really heterotic superstring theory. It has a huge set of symmetries, including the conformal string symmetries that encompass the bootstrap principle and general relativity, and an internal symmetry group E_8 x E_8 that undergoes spontaneous symmetry breaking. The outer layers of the onion show gravitation and electromagnetism. Penetrating a little further turns up SU_2 x U_1, SU_3 of color, and the bootstrap idea. And so it goes on. The mathematics at each level is not usually identical with that at the next level, but it has a strong family relationship. The successive renormalizable approximate theories, by the way, represent autonomous shells that depend on what is inside only through the renormalized parameters.

At the modest level of earlier science, this sort of self-similarity is strikingly apparent. Electricity, gravitation, and magnetism all have the same $1/r^2$ force, and Newton, as we have seen, suggested that there might be some short-range forces as well. Perhaps in some lost manuscript he proposed the Yukawa potential!

Now that scientists and mathematicians are paying attention to scaling phenomena, we see in the study of complex systems astonishing power laws extending over many orders of magnitude. Often the underlying mechanism is changing while the power law still holds, as for the cosmic ray energy spectrum, the advance of technologies over time, and so forth.

The renormalization group, which we invented for renormalizable quantum field theory, turns out to apply not only to critical phenomena in condensed matter, but to numerous other far-flung subjects as well.

The biological and social sciences are just as much

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The Simply Complex: Trendy Buzzword or Emerging New Science?

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by John Casti

A few years ago, I saw a cartoon showing two scientists arguing over the the meaning of complexity. In suitably dogmatic terms the first scientist asserted, "Complexity is what you don't understand." Responding to this temerarious claim, his colleague replied, "You don't understand complexity." This circular exchange mirrors perfectly to my eye how the informal term "complexity" has been bandied about in recent years—especially within the normally flinty-eyed community of system scientists—as a characterization of just about everything from aardvarkology to zymurgology. Without benefit of anything even beginning to resemble a definition, we find the putative "science" of complexity being described in terms rosy enough to emit heat: *adaptive* behavior, *chaotic* dynamics, *massively parallel* computation, *self-organization,* and even on to the *creation of life* itself within the cozy confines of a machine. And to add a final touch of spice, all of this hoopla often comes wrapped up in language vague enough to warm the heart of any Continental philosopher. But useful as all this fuzziness is for fending off cocktail-party bores and writing research grant proposals, it becomes a major impediment when we start talking seriously about a "science" of complex systems. The problem is that an integral part of transforming complexity (or anything else) into a science involves making that which is fuzzy precise, not the other way around, an exercise we might more compactly express as "formalizing the informal." This short essay represents an exploration into some of the dimensions of this problem, as we try to "scientify" the simply complex.

Still More Complex

The science-fiction writer Poul Anderson once remarked, "I have yet to see any problem, however complicated, which, when you looked at it the right way, did not become still more complicated." Substituting the word "complex" for "complicated," this statement serves admirably to capture the two key points needed to understand what's at issue in turning the casual, everyday notion of a complex system into something resembling an actual science.

> ...complexity is an inherently subjective concept; what's complex depends upon how you look at it...complexity resides as much in the eye of the beholder as it does in the structure and behavior of a system itself.

The first is the realization that complexity is an inherently subjective concept; what's complex depends upon how you look. So when we speak of something being "complex," what we're really doing is making use of everyday language to express a feeling or impression that we characterize by the label "complex." But the meaning of something depends not only upon the language in which it is expressed (i.e., the code), the medium of transmission, and the message, but also upon the context. In short, meaning is bound up in the whole

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involved in these discoveries of scaling behavior as the physical sciences. We are always dealing with Nature consonant and conformable to herself, not only within scaling behavior but also in the occurrence of similar phenomenological laws in a plethora of disparate areas. So the approximate self-similarity of the laws of nature runs the gamut from the simple underlying laws of fundamental physics to the phenomenological laws of the most complex phenomena. No wonder our mathematics keeps working so well in the sciences, when self-similarity is so widespread.

Of course there may be something important here about the nature of mathematics itself. In connection with that, let me close by paraphrasing some wonderful remarks made by that brilliant and modest theoretical physicist Feza Gürsey on the occasion of his receiving, at the University of Miami, not the valuable kind of prize he deserves, but half of the $1,000 Oppenheimer Prize.

He said, more or less, that he had achieved some success by pointing, often before other theorists, to mathematical structures that would be useful in the near future in elementary particle physics. But often he hadn't had any clear idea of exactly how or why these mathematical methods would be used. He compared himself with Inspector Clouseau, bumbling along, bumping into walls, but somehow finally pointing to the right suspects. Why, he asked, did the Inspector Clouseau method work? Maybe, he suggested, because such mathematical structures are comparatively rare, so that it is possible to find and identify something like the exceptional group E_8 as an object of interest simply because structures with its remarkable properties are not thick on the ground. Thus it may be that the character of *mathematics* plays a role in our story, along with Nature consonant and conformable to herself.